MODELING AN INPUT-OUTPUT GEOKINETIC SYSTEM UTILIZING A FINITE ELEMENT APPROACH

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MODELING AN INPUT-OUTPUT GEOKINETIC SYSTEM
UTILIZING A FINITE ELEMENT APPROACH

by

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March 1975

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Modeling an Input-Output Geokinetic System
Utilizing a Finite Element Approach

by

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Submitted in partial fulfillment of the
requirements for the degree of

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I. INTRODUCTION

This study explores modeling a specific input-output system in the field of geokinetics. A finite element technique is utilized as the system simulator. The Earth's surface movements are the output to several geophysical phenomena. One of the most significant geophysical inputs is ocean tital loading. Finite element modeling provides an effective technique for exploring the detailed surface movements of the earth in response to ocean loading.

The first chapter of this paper discusses the bounds of the finite element model. The finite element model is contrasted with an analytical half space Boussinesq technique in order to provide the level of detail which should be included in the finite element model. Tilts resulting from two and three dimensional surface loads on the Boussinesq half space models bracket the tilt produced by the two dimensional finite element model. In the second chapter relationships are explored between surface loads and fault zone tilt response as a function of fault zone shear strength, and distance of the fault from the load. The results indicate a logarithmic relationship between the magnitude of fault zone tilt and distance of the fault from the load. The final chapter describes four profiles of the California coastline in an attempt to relate real data collected by the United States Geological Survey to model results.

It is assumed that the reader is already familiar with solving families of differential equations by finite difference techniques and their applications to geophysical sciences. Only token consideration is given in this paper to the theory of finite difference techniques and their application to finite element modeling. The reader unfamiliar with finite element methodology is encouraged to read the appendix of this paper containing the documentation of the finite element computer program. This documentation should give the reader a better perception of the finite element modeling technique. There are also several excellent references published on the subject of finite element techniques [1, 9, 15].

There are many applications of the finite element technique besides solving geophysically related systems. The pure research which is presented in this paper can be applied to problems not only in earthquake research but, for example, also in air defense missile systems [2]. Accurately predicted ground motion near siloed missiles can be programmed into the missile's prelaunch parameters of the inertial guidance system. This information results in fewer course corrections required for the missile and therefore a higher probability of hitting its target.

The finite element computer program utilized in this paper was especially designed for analyzing deformations and strains resulting from a system of loads applied to a structure. This two dimensional finite element computer program was developed at the National Center for Earthquake Research of the United

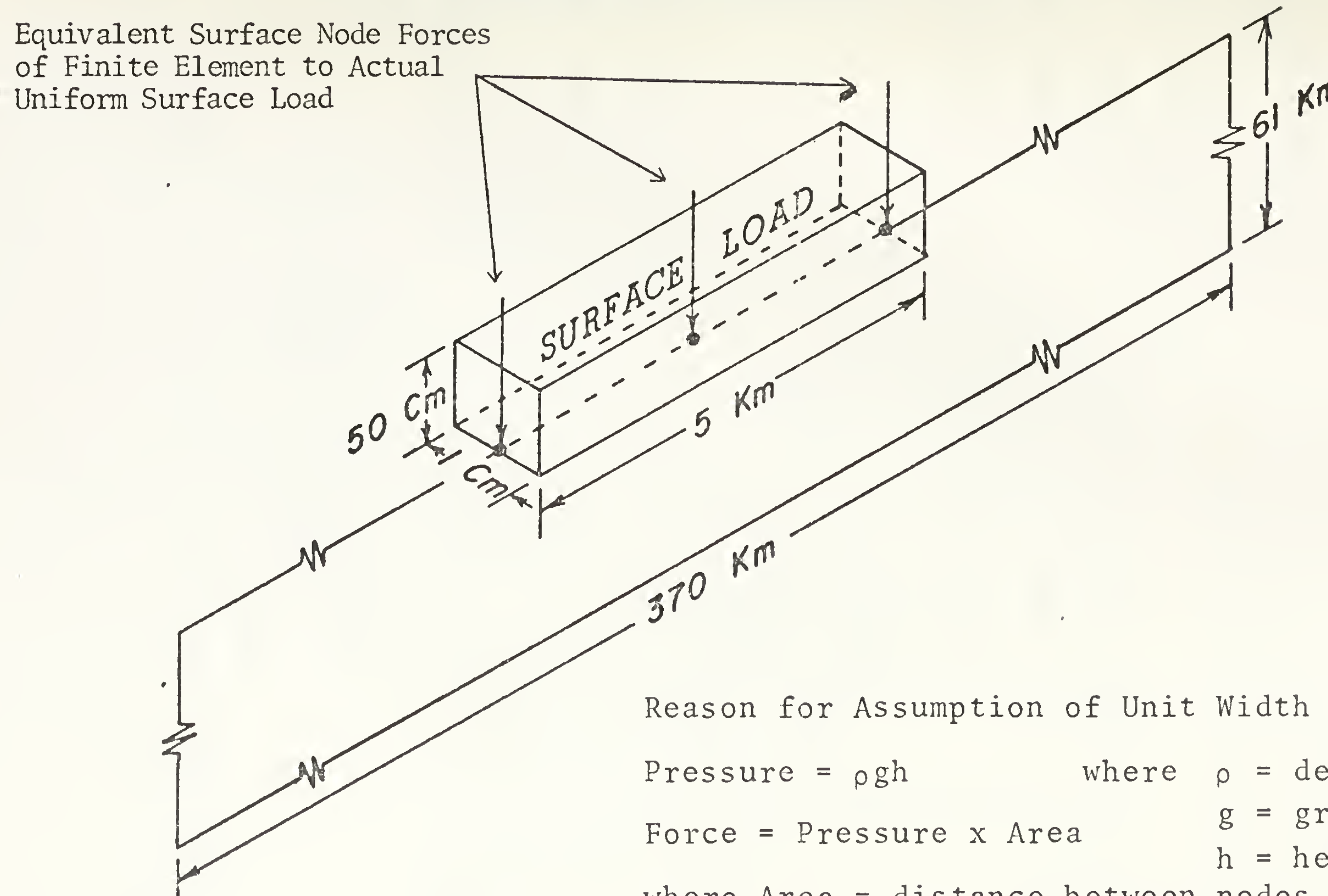
States Geological Survey by Dr. James H. Dieterich [3], Dr. M. Darroll Wood and refined by the author. The author is greatly indebted to Dr. M. Darroll Wood for his cooperation, guidance and geophysics expertise which made this paper possible. The terminology and notation used in this paper was adopted from Dr. Wood's doctorial thesis [12]. The author also expresses his appreciation to Professor Donald Barr and Professor Rex Shudde for their helpful criticism and ideas during the stages of development of this paper. The William R. Church Computer Center was a tremendous asset in the cooperation and service which it provided in processing the finite element computer programs.

II. FINITE ELEMENT MODEL CONTRASTED WITH BOUSSINESQ MODEL

In 1878, Boussinesq calculated the vertical deviation of an elastic plane surface of a homogeneous medium under the loading effect caused by water. He showed that the vertical displacement of the Earth's crust was proportional at each point to the gravitational potential of the load and was a function of the elastic constants of the medium [8]. The Boussinesq technique has long been used as a simple approximation of load tilt. Load tilt is principally the gradient of the displacement field of the Earth's surface and two other secondary effects. The principal effect and the one which the finite element model simulates is that of the variable flexure of the Earth's crust under the loading effect caused by the oceans. The other effects are the attraction of the masses of water causing a vertical displacement of the Earth's surface, and the variation of the potential due to secondary deformation of the Earth's crust, an effect which is opposite in direction to the first and second effects [8].

The finite element structure is a two dimensional model in which the load is assumed to have unit width (Fig. 1). For this reason, it is difficult to directly compare the finite element model with Boussinesq half space models. The tilts calculated from the Boussinesq technique bounded the tilt produced from the finite element model in each of six

Finite Element Structure and Load



Reason for Assumption of Unit Width of Surface Load

Pressure = ρgh where ρ = density of water

g = gravity

Force = Pressure x Area

h = height of load

where Area = distance between nodes (length of load) multiplied by a unit depth.

FIGURE 1

cases processed with different loads and elastic parameters. The finite element model overestimated the tilt calculated from the equivalent Boussinesq model with a two dimensional load, and consistently underestimated the tilt calculated from the equivalent Boussinesq model with a three dimensional load. The following description is of one model processed by the finite element model and the calculations associated with the Boussinesq method.

The Boussinesq method describes load tilt δ_x^{ℓ} as the deviation of the vertical $\delta_x^a(r)$ due to a load of radial extent r multiplied by the Boussinesq factor m. Thus:

$$\delta_x^{\ell} = \delta_x^a(r) \cdot m \text{ where } m = \frac{\lambda + 2\mu}{4\pi(\lambda+\mu)\mu} \cdot \frac{g^2}{G}$$

where μ = shear modulus
λ = modulus of rigidity (Lamé Constants)
g = gravity
G = gravitational constant.

The deviation of the vertical $\delta_x^a(r)$ due to a load of extent r and height h is the ratio of the horizontal gravity g_x to the vertical gravity attraction [12]. For the three dimensional Boussinesq model $\delta_x^a(r) = \frac{G\rho h}{g} \frac{d}{dx} \int \frac{dr}{r} = \frac{G\rho h}{g} \ln(r)$

where ρ = density of load (water)
G = gravitational constant
h = height of the load
g = gravity (vertical)
r = radial extent of load.

For the two dimensional Boussinesq model $\delta_x^a(r)$ is computed using a line integral. Thus, $\delta_x^a(r) = \frac{G\rho h}{g \cdot r}$ where ρ, G, h, g, r are the same as above [12].

The finite element model was programmed to use the same inputs as the Boussinesq models of a homogeneous medium

containing elastic parameters $\lambda = \mu = 6.2 \times 10^{11}$ dynes/cm^2 (Lamé constants).

To simulate the Boussinesq boundary conditions, the bottom edge of the finite element structural model was specified as rigid. The load was a symmetrical surface load of water one centimeter in height and five kilometers in length.

The tilt calculated at the load center of the finite element model was 3.85×10^{-10} radians. The three dimensional Boussinesq method produced a load tilt of 2.47×10^{-9} radians, and the two dimensional Boussinesq methods produced a load tilt of 3.77×10^{-16} radians.

Because of the consistency with which the finite element model was bracketed by the Boussinesq methods, it is presumed that the finite element program will be a useful tool for detailed analysis of deformations resulting from a variety of loads applied to inhomogeneous structures. In order to have continuity throughout this paper, the bottom edges of remaining finite element models were structurally designed rigid.

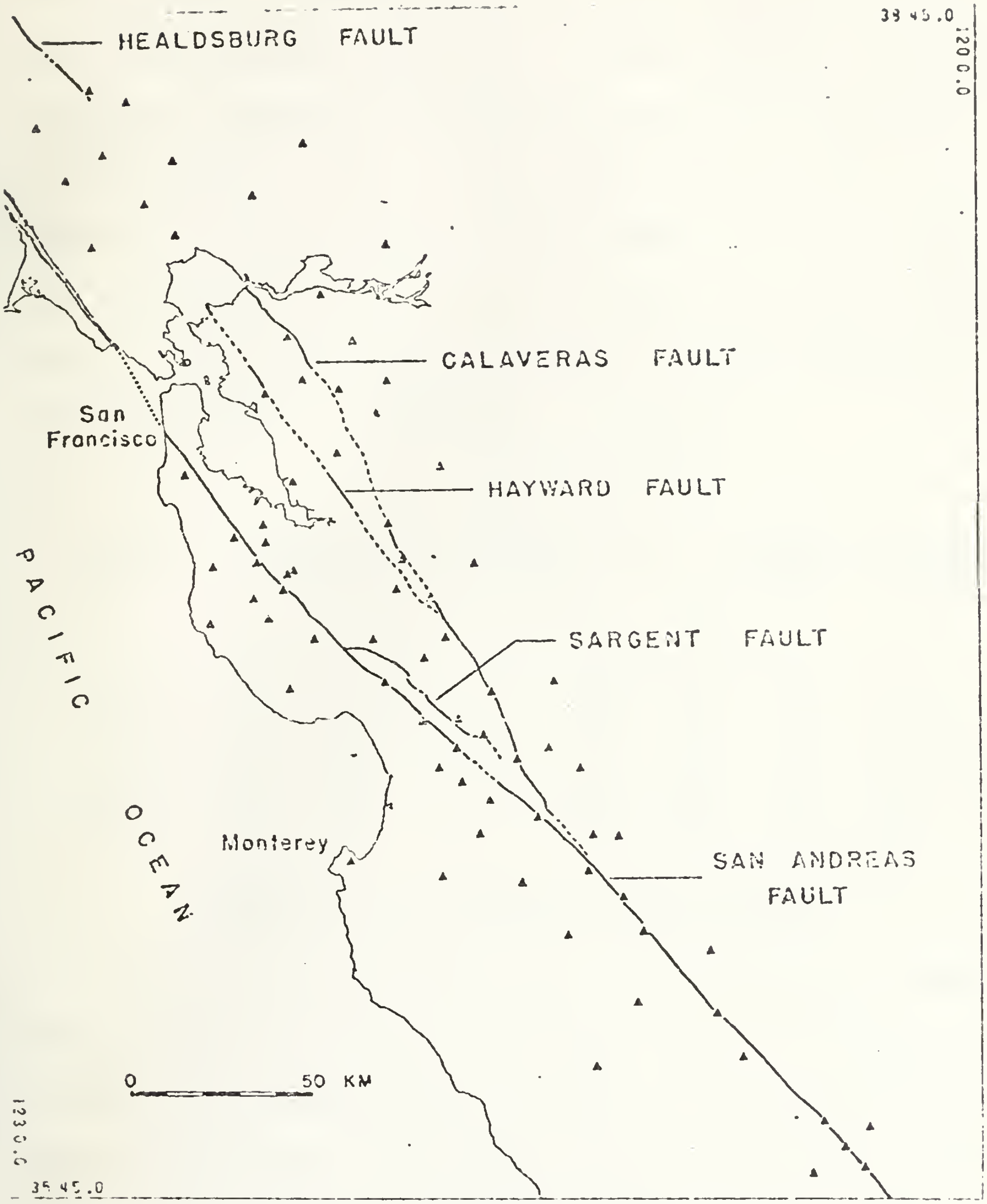
III. DEVELOPMENT OF RELATIONSHIPS BETWEEN FAULT ZONE TILT, FAULT LOCATION, FAULT SHEAR STRENGTH AND SURFACE LOADS

The response of earth structures to surface loads was explored utilizing the deterministic finite element model. The finite element model simulates the behavior of a discontinuous structure under assymmetric loading. The model structure was matched to the real Earth structure by laterally and vertically varying the known inhomogeneities of the earth. The response of the region near a vertical discontinuity, namely a fault, was of particular interest. The smallest homogeneous elements of the model were assumed to behave elastically for small increments of stress. However, the shear that is distributed in and near the fault zone elements or cells in the finite element grid may have varied considerably due to the contrast in Lamé constants for adjoining elements. The large difference in shear produces steps in the displacement field and therefore poses a problem of smoothness.

Tilt measurements in borehole and observatory sites have been made along the California coastline. Tilt measurements at tidal sensitivities (10^{-8} μ radians) show the Earth's surface response to oceanic loading is greater than responses from any other continuous source [13]. Conceivably, ocean loading constitutes a forcing function that can be used to derive the bulk elastic properties of the Earth. However, it has been shown that ocean tidal spectra are non-stationary

in all but lunar frequencies. The lunar semi-diurnal wave (M_2) is theoretically 85% of the energy of the total theoretical tidal spectrum [8]. In actuality this is not true. However, the M_2 line is sufficiently energetic and sufficiently removed from the contaminated tidal spectrum to be used as a stationary forcing function for earth response studies. For these reasons, the model surface load corresponded to the amplitude of the M_2 frequency. A value of fifty-three centimeters for the M_2 amplitude was chosen to represent the ocean loading in the central and northern California region [14]. In order to remain within the required aspect ratio in each cell of the finite element grid of 10:1 and to account for approximately 95% of the ocean loading effect as calculated by Boussinesq, the length of the load was chosen to be 165 kilometers [8]. With this loading, the investigation was limited to exploring relationships between fault zone tilt, fault shear strength, and the distance from the load to the fault. A factorial arrangement was developed by processing three slightly different velocity structure models having twelve fault locations and four fault shear moduli. The width and depth of the fault zone were set at five and sixteen kilometers respectively. These dimensions conform with Mayer-Rosa's [7] conclusion that the fault zone must be a low velocity zone at least several kilometers wide, extending into the lower crust. The fault zone was shifted consecutively in five kilometer increments from the edge of the load (coastline) to 60 kilometers inland.

This range of distances spans the extent of separation between the San Andreas Fault and the Pacific Ocean throughout most of central and northern California (Fig. 2). At each fault zone location, the shear strength of the fault was varied by adjusting the shear modulus in the fault zone. Starting with values of fault zone shear modulus set equal to adjacent elements, and then in consecutive models reducing the shear modulus in the fault zone one order of magnitude decrements, a maximum difference of four orders of magnitude less of shear strength in the fault zone was achieved.



Location map of U.S. Geological Survey seismograph stations and major faults in central California.

FIGURE 2

The Lamé constants computed for each cell were derived from studies of the seismic velocity models of Stewart [11] and the works he referenced. Three slightly different velocity structures were developed to observe differences in fault zone tilt response. The Poisson ratio was adjusted from 0.4 at the surface of the model to 0.25 at depths below ten kilometers. The density was derived using a Nafe-Drake curve. The three velocity models are represented by their associated elastic parameters in Table I.

TABLE I

DEPTH	HARD BOTTOM MODEL		SOFT BOTTOM MODEL		STEWART MODEL GABILAN		STEWART MODEL DIABLO	
	λ	μ	λ	μ	λ	μ	λ	μ
1.0	0.28	0.12	0.28	0.12	0.28	0.12	1.1	4.5
6.0	2.3	1.5	2.3	1.5	1.6	1.0	2.3	1.5
16.0	3.3	3.3	3.3	3.3	3.3	3.3	3.3	3.3
26.0	4.3	4.3	4.3	4.3	4.3	4.3	4.3	4.3
41.0	6.5	6.5	6.5	6.5	6.5	6.5	6.5	6.5
61.0	6.5	6.5	1.1	0.45	6.5	6.5	6.5	6.5

λ and μ are in units of 10^{11} dynes/cm^2.
Depths are in units of kilometers.

Tilt was calculated as the maximum positive slope of displacement throughout each fault zone. The phase of tilt in which the displacement gradient is toward the load is taken as positive. By examining the results of the finite element model for these three velocity structures, a relationship between fault zone tilt and the distance from the load to the fault was hypothesized. The calculated tilt in the fault zone was plotted logarithmically against linear distance between the fault zone and the edge of the load (Fig. 3, 4, 5). For distances between 10 and 45 kilometers,

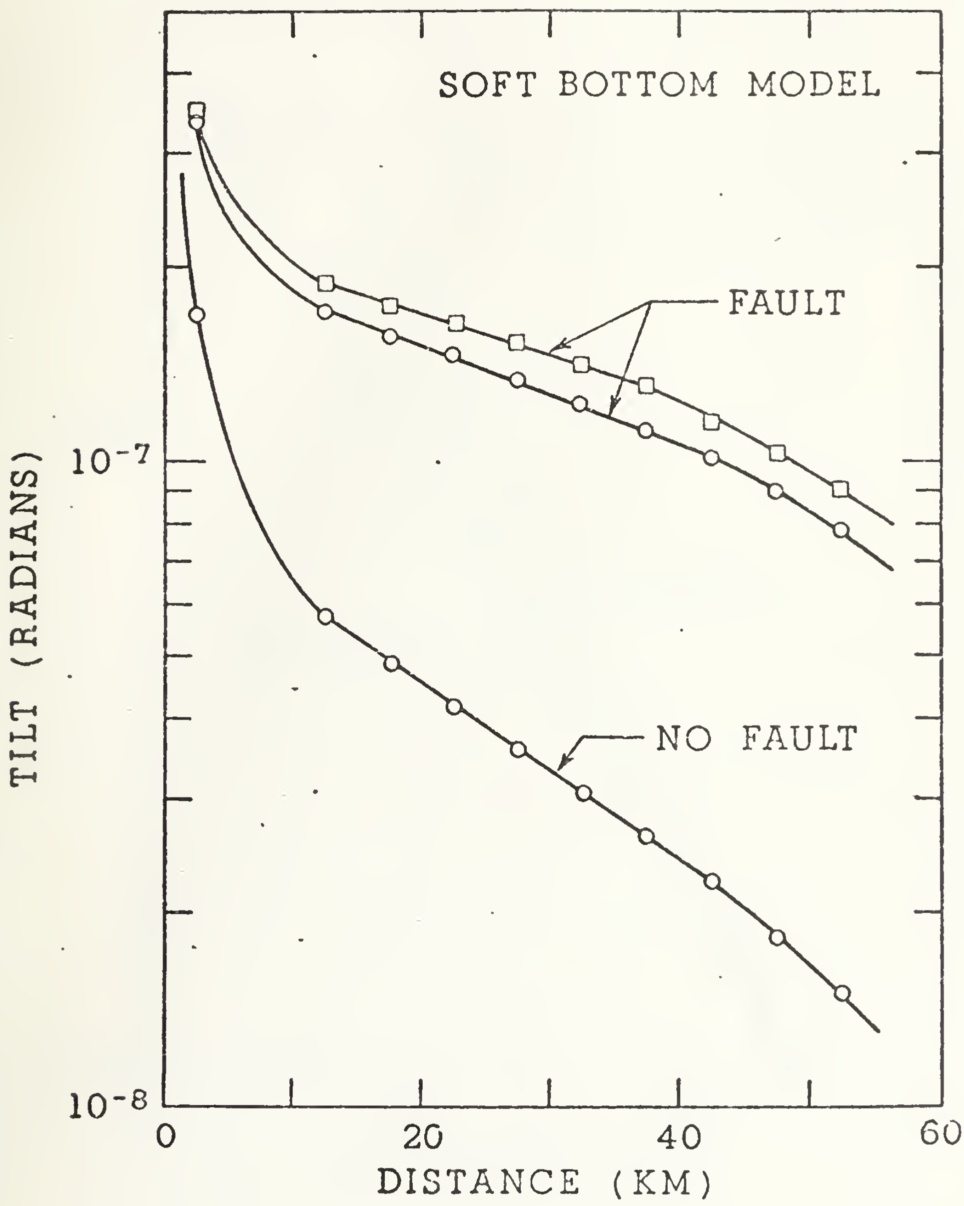

SOFT BOTTOM MODEL
FAULT
NO FAULT
TILT (RADIANS)
10^{-7}
10^{-8}
0
20
40
60
DISTANCE (KM)

FIGURE 3

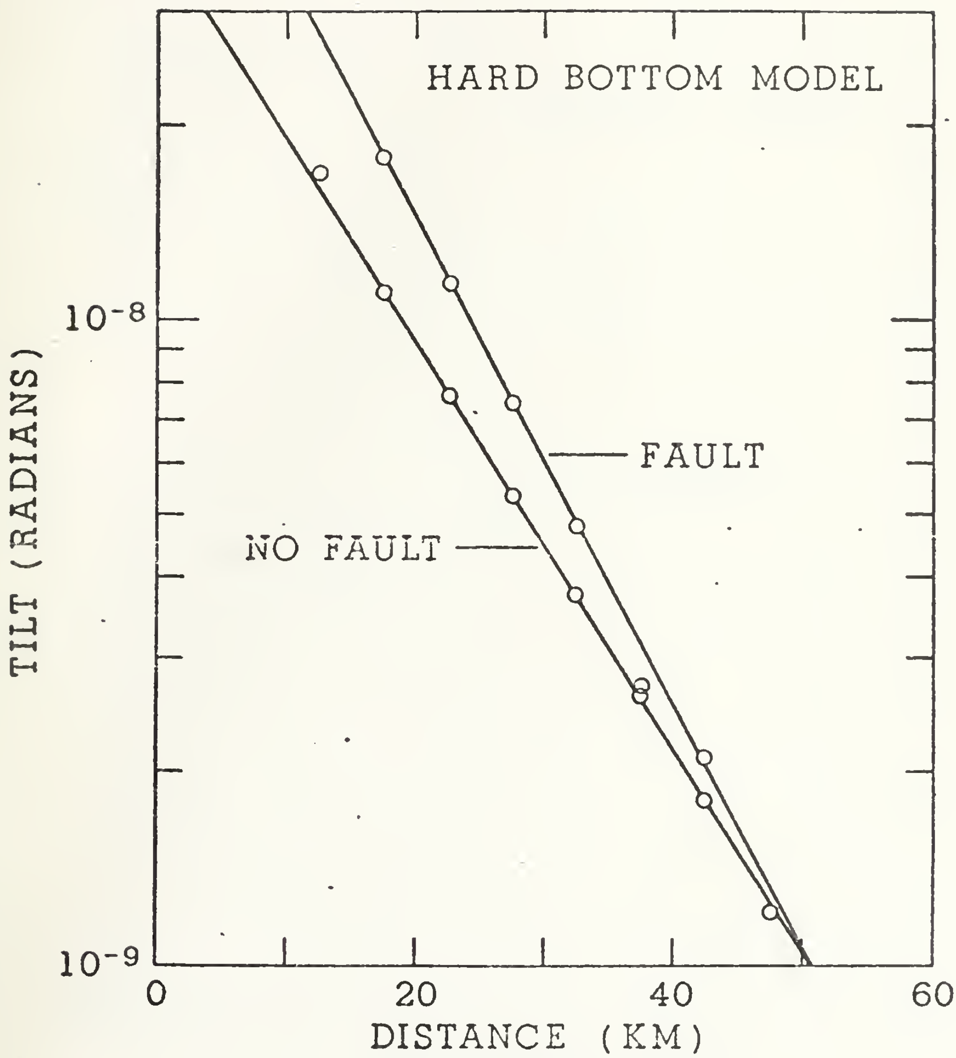
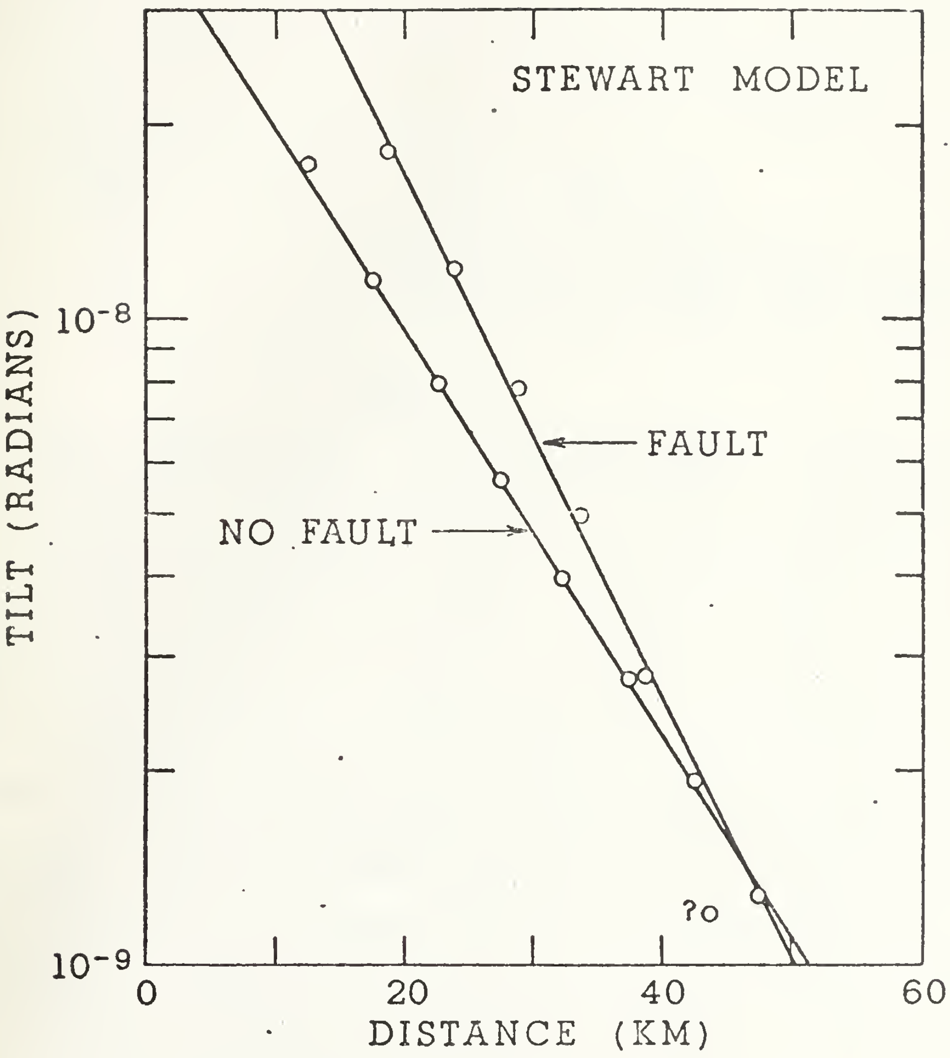

HARD BOTTOM MODEL
FAULT
NO FAULT
TILT (RADIANS)
10^{-8}
10^{-9}
0
20
40
60
DISTANCE (KM)

FIGURE 4



STEWART MODEL
FAULT
NO FAULT
TILT (RADIANS)
10-8
10-9
?o
0
20
40
60
DISTANCE (KM)

FIGURE 5

the tilt measurements lie nearly on a straight line for all three models and fault moduli. Straight lines on semi-logarithmic graph paper characterize the relationship as exponential. Equations describing the relationship between fault zone tilt and fault zone location are easily estimated from the graphs. The basic model is shown below, followed by Table II showing the value of each of the parameters for each velocity structure and fault shear strength.

$T = T_o e^{-s \cdot d}$ where T = fault zone tilt
T_o = tilt intercept
s = slope of straight line
d = distance between fault zone and edge of the load (coastline)

TABLE II

	SOFT BOTTOM MODEL		HARD BOTTOM MODEL		STEWART MODEL	
	NO FAULT	FAULT	NO FAULT	FAULT	NO FAULT	FAULT
T_o	8.5×10^{-8}	2.15×10^{-7}	3.9×10^{-8}	8.0×10^{-8}	4.0×10^{-8}	1.1×10^{-7}
S	0.032	0.018	0.073	0.087	0.072	0.094
RANGE	$5 < d < 50$ km		$10 < d < 50$ km		$10 < d < 45$ km	

Tilt is strongly dependent on the shear modulus of the fault zone. When the shear modulus is decreased in the fault zone one order of magnitude from surrounding material, there is a distinct difference in the resulting tilt as compared with no fault structures. However, decreasing the shear modulus in the fault zone consecutively two, three and four orders of magnitude resulted in a small change of the fault zone tilt. This apparent convergence can best be seen from the table of calculated tilt in the fault zone for the three models (Table III). Plots of the shear modulus (μ) versus

TABLE III

FAULT ZONE	SOFT BOTTOM MODEL NO FAULT	$\mu=10^9$	$\mu=10^8$	$\mu=10^7$	HARD BOTTOM MODEL NO FAULT	$\mu=10^9$	$\mu=10^8$	$\mu=10^7$	STEWART MODEL NO FAULT	$\mu=10^9$	$\mu=10^8$	$\mu=10^7$	$\mu=10^6$
0-5	25.2	40.7	41.8	41.9	20.8	40.1	42.6	42.8	23.1	45.5	47.5	47.6	47.7
5-10	7.24	19.6	21.0	21.2	2.99	8.90	9.29	9.33	3.23	10.1	10.6	10.7	10.7
10-15	5.72	17.0	18.7	18.8	1.69	3.20	3.27	3.28	1.73	3.77	3.89	3.90	3.90
15-20	4.84	15.6	17.3	17.5	1.11	1.77	1.88	1.89	1.14	1.82	1.89	1.90	1.90
20-25	4.17	14.5	16.3	16.5	0.76	1.14	1.24	1.26	0.79	1.18	1.24	1.25	1.25
25-30	3.56	13.5	15.2	15.4	0.53	0.75	0.84	0.85	0.56	0.78	0.83	0.84	0.84
30-35	3.08	12.4	14.0	14.2	0.37	0.48	0.54	0.55	0.40	0.50	0.53	0.54	0.54
35-40	2.62	11.2	12.8	12.9	0.26	0.27	0.32	0.33	0.28	0.28	0.31	0.31	0.31
40-45	2.21	10.1	11.5	11.7	0.18	0.11	0.15	0.16	0.19	0.12	0.13	0.13	0.13
45-50	1.83	8.93	10.3	10.4	0.12	.004	0.02	0.03	0.13	*	.0004	.0004	.002
50-55	1.51	7.82	9.01	9.15	0.08	*	*	*	0.08	*	*	*	*
55-60	1.20	*	*	*	0.05	*	*	*	0.05	*	*	*	*

Table contains tilt calculated in each fault zone location of three velocity structure models.

Tilt units (10^{-8} radians).

Fault zone is the distance from the edge of the load to fault zone (kilometers).

* Phase of tilt vector inverted.

fault zone tilt for each of the fault zone locations and velocity structures are shown in figures 6, 7, 8.

The dotted line segments of the graphs are the interesting feature. This part of the graph depicts the relationship between a no fault structure and a structure containing a fault zone of shear modulus decreased by one order of magnitude from surrounding material. There appears to be no similarities in this portion of the graphs for different fault zone locations. The dotted line portion of the graphs are flatter at fault zone locations further away from the load. The inference is that the more distant the loads, the less sensitive the response to a local discontinuity. Also, greater depths are required to adequately model distant loads. If the dotted lines in the graph are approximated as straight lines, then an equation relating the shear modulus to fault zone tilt is given by

$T = A \cdot U^{-n}$, where T = fault zone tilt
A = tilt intercept
U = shear modulus of fault zone
n = slope of the straight line.

However, further study into particular fault zone locations is required before this equation can be verified.

There are several shortcomings in the modeling which has been presented. First, the soft bottom model is not a realistic representation of the Earth's velocity structure. However, this model does show a sensitivity with which fault zone tilt varies with slight changes in the overall velocity structure. The soft bottom model also unmistakably has the characteristic exponential relationship between fault zone

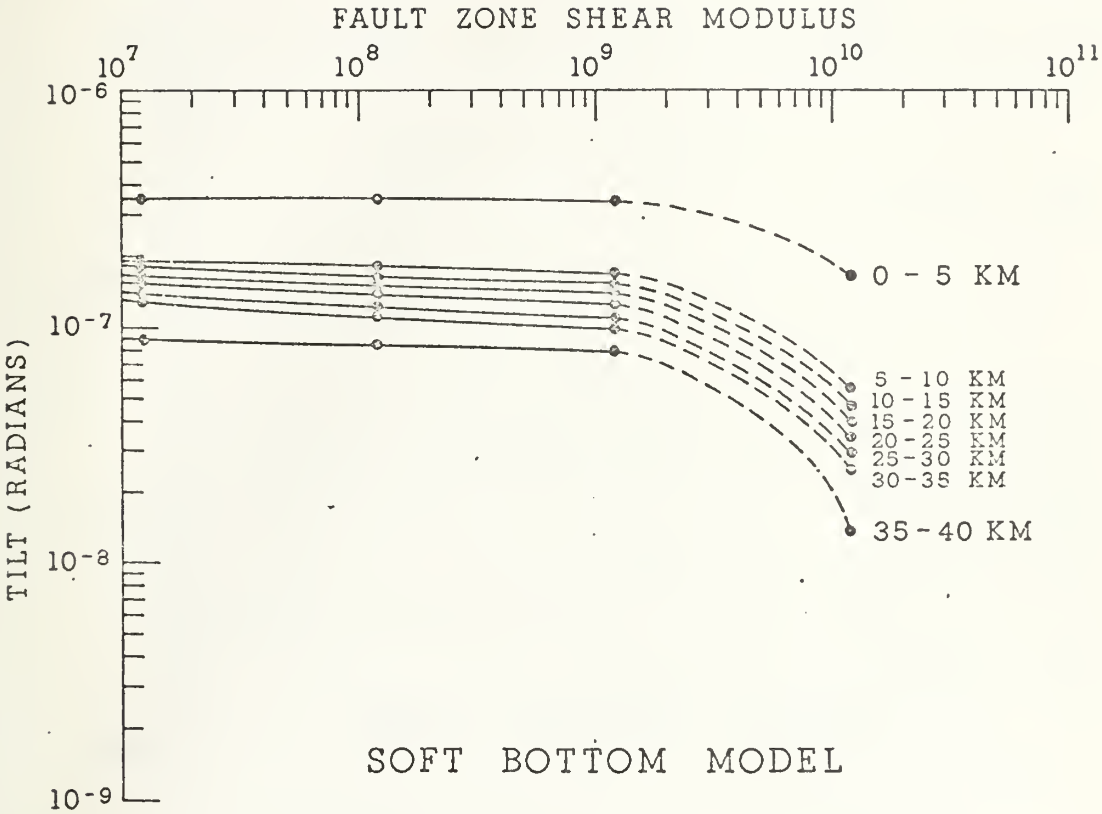

FAULT ZONE SHEAR MODULUS
10^7
10^8
10^9
10^10
10^11
10^-6
10^-7
10^-8
10^-9
TILT (RADIANS)
0 - 5 KM
5 - 10 KM
10 - 15 KM
15 - 20 KM
20 - 25 KM
25 - 30 KM
30 - 35 KM
35 - 40 KM
SOFT BOTTOM MODEL

FIGURE 6

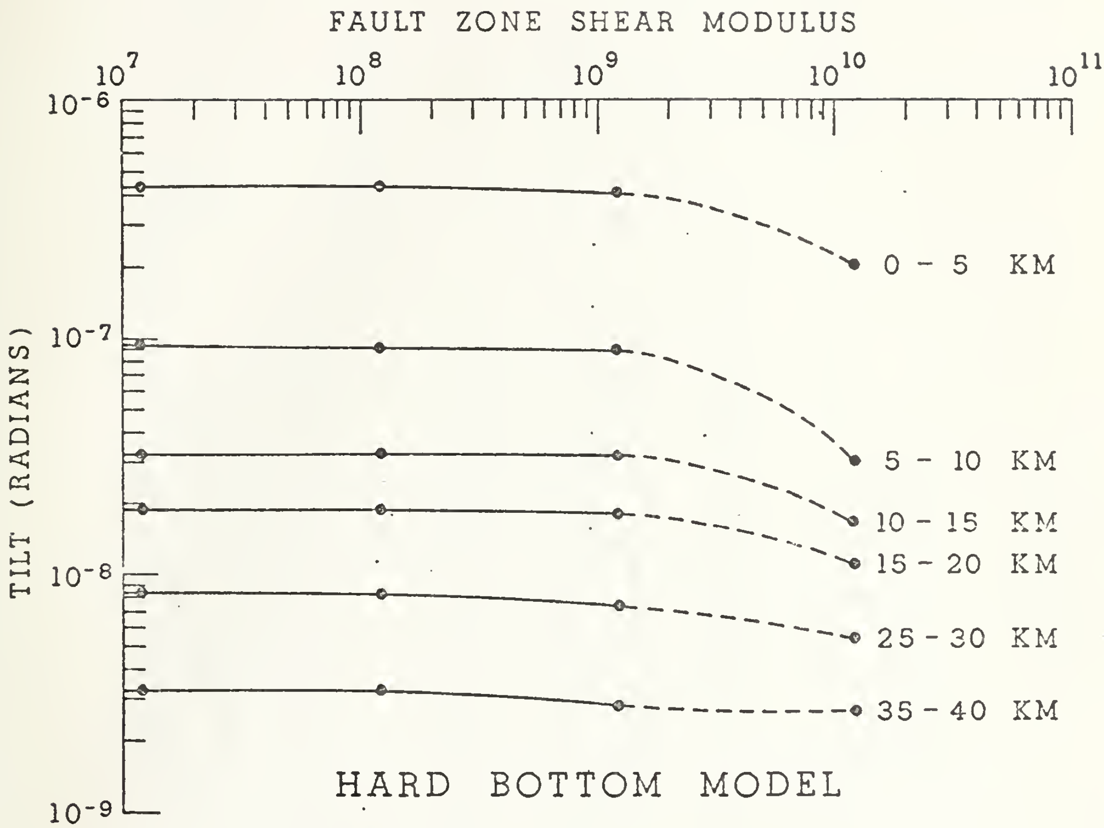

FAULT ZONE SHEAR MODULUS
10^7
10^8
10^9
10^{10}
10^{11}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
TILT (RADIANS)
0 - 5 KM
5 - 10 KM
10 - 15 KM
15 - 20 KM
25 - 30 KM
35 - 40 KM
HARD BOTTOM MODEL

FIGURE 7

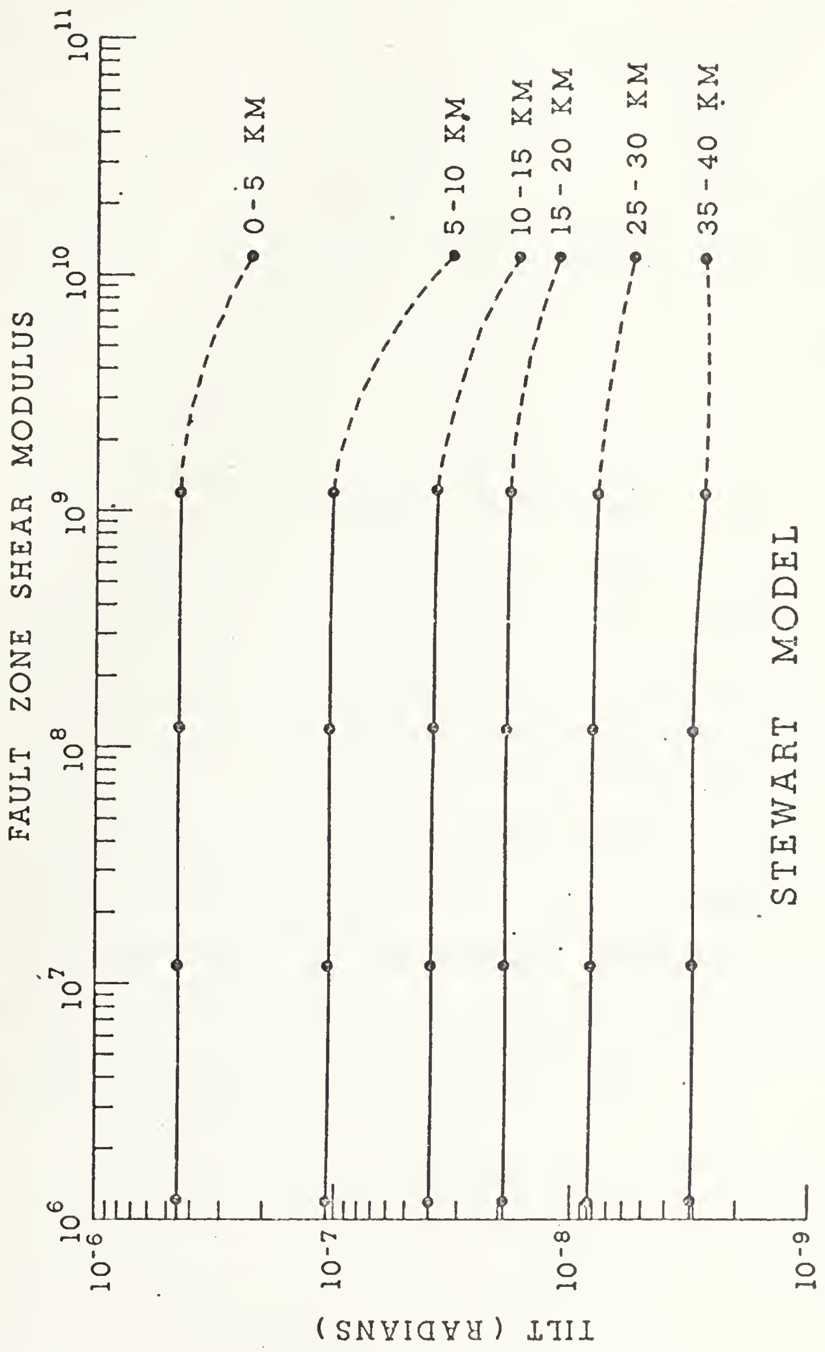

FAULT ZONE SHEAR MODULUS
10^6
10^7
10^8
10^9
10^{10}
10^{11}
TILT (RADIANS)
10^{-6}
10^{-7}
10^{-8}
10^{-9}
0-5 KM
5-10 KM
10-15 KM
15-20 KM
25-30 KM
35-40 KM
STEWART MODEL

FIGURE 8

tilt and fault zone location in the range of 10 to 45 kilometers. Both the hard bottom model and the Stewart model are better models of the real Earth's velocity structure [11]. However, because of grid design constraints, the detail of the velocity structure which could be represented was limited, especially in the 0 to 15 kilometer depth range. A plot of the displacement field throughout the structure revealed a final shortcoming of the grid design. The depth used in the present model is too shallow to explore fault zone locations farther than 50 kilometers from the edge of the load. It is apparent from figure 9 that the problem of grid design is directly related to the boundary conditions of the rigid bottom edge. The rigid bottom has the tendency to make the next to bottom layer of the structure oscillate about zero centimeters displacement at distances over 50 kilometers from the load. Also, the ratio of effective distance of the load from a response point to the depth used in the model should not exceed unity. This deduction is demonstrated in figure 9 where the depth of the model structure is 61 kilometers and invalid results occurred beyond the region of 50 to 60 kilometers from the edge of the load.

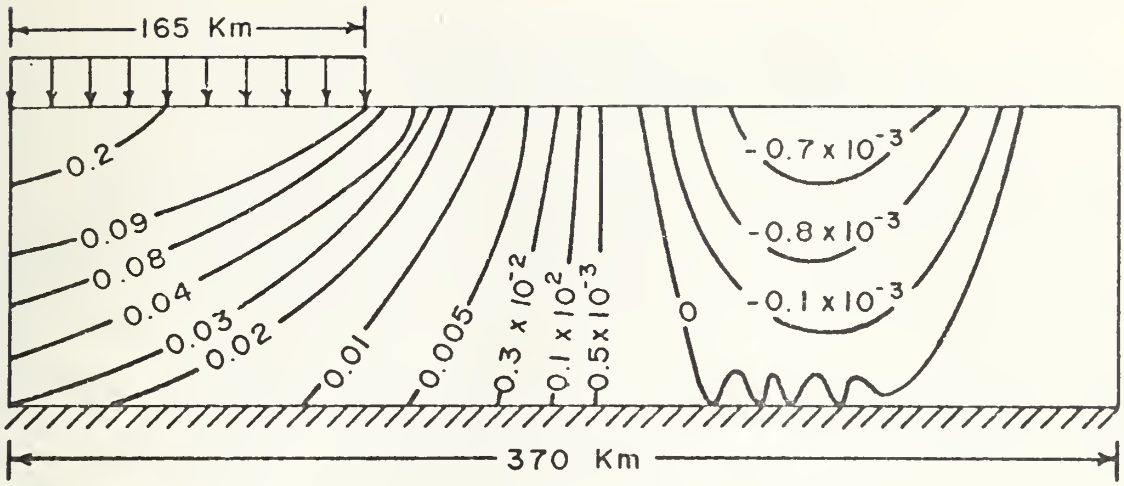


Figure of Displacement Field in the Hard Bottom Model

FIGURE 9

IV. MODELS OF FOUR PROFILES OF THE CALIFORNIA COASTLINE

Four profiles of the California coastline were modeled using the finite element technique in an attempt to compare the results with data collected by the United States Geological Survey. However, the results of these investigations point out pitfalls in the modeling phase. The four profiles are pictured in Figure 10. For reasons given below, the only useful profile is profile A. Figure 11 shows the displacement of the surface nodes plotted against distance. Figure 11 also depicts the location of the loading and the location of the fault zone. The upper dotted line results when a fault zone shear modulus is one order of magnitude less than the shear modulus of adjacent surface elements. The lower curve is based on the assumption that no fault zone exists and the elements in the fault zone are therefore identical with those adjacent to the zone at the same depth. The fault zone tilt associated with the no fault structure was 4.18×10^{-7} radians whereas the fault zone curve has a calculated tilt of 2.09×10^{-5} radians. This tilt was expected from the graphs developed in the previous chapter and is comparable to actual tilt data in the San Francisco Bay area. Figures 12, 13, 14 display the same attributes for profiles B, C, and D as that of Figure 11 of profile A. In profiles B, C, and D the ocean loading was not sufficient in magnitude to accurately model ground movement in these profiles. This resulted in the instabilities

discussed in the previous chapter. At least 165 kilometers or more of ocean loading is required to provide an adequate forcing function for the modeling of all profiles. Zones of 2.5 kilometers in width are probably too small to be realized by the existing grid design. Therefore, fault zones of 2.5 kilometers in width showed little or no change in the displacement field between no-fault and fault structures. Also, as stated in the previous chapter, the grid design is too shallow to place fault zones farther than 50 kilometers from the edge of the load and receive valid information.

FOUR CALIFORNIA PROFILES

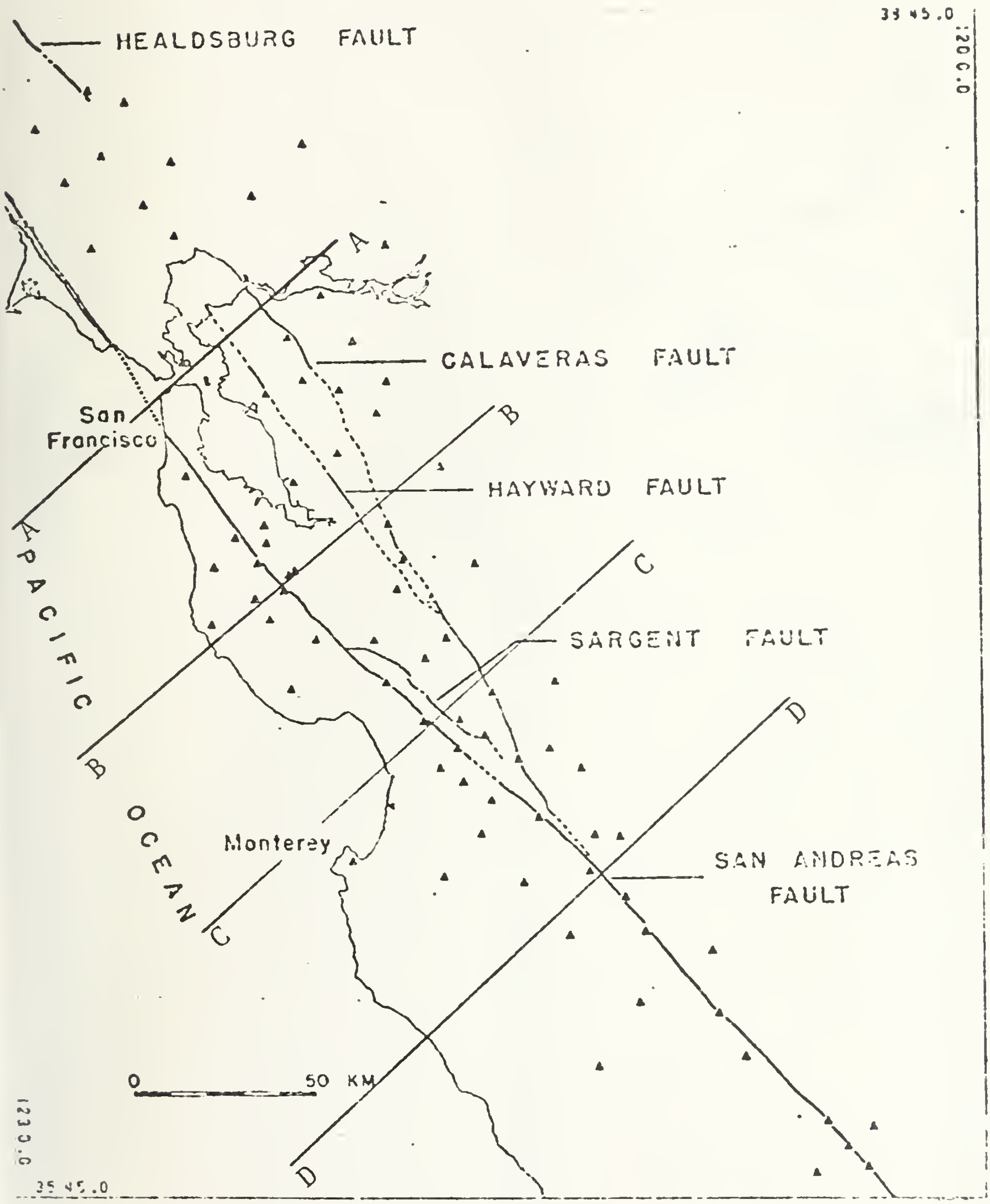


FIGURE 10

PROFILE A

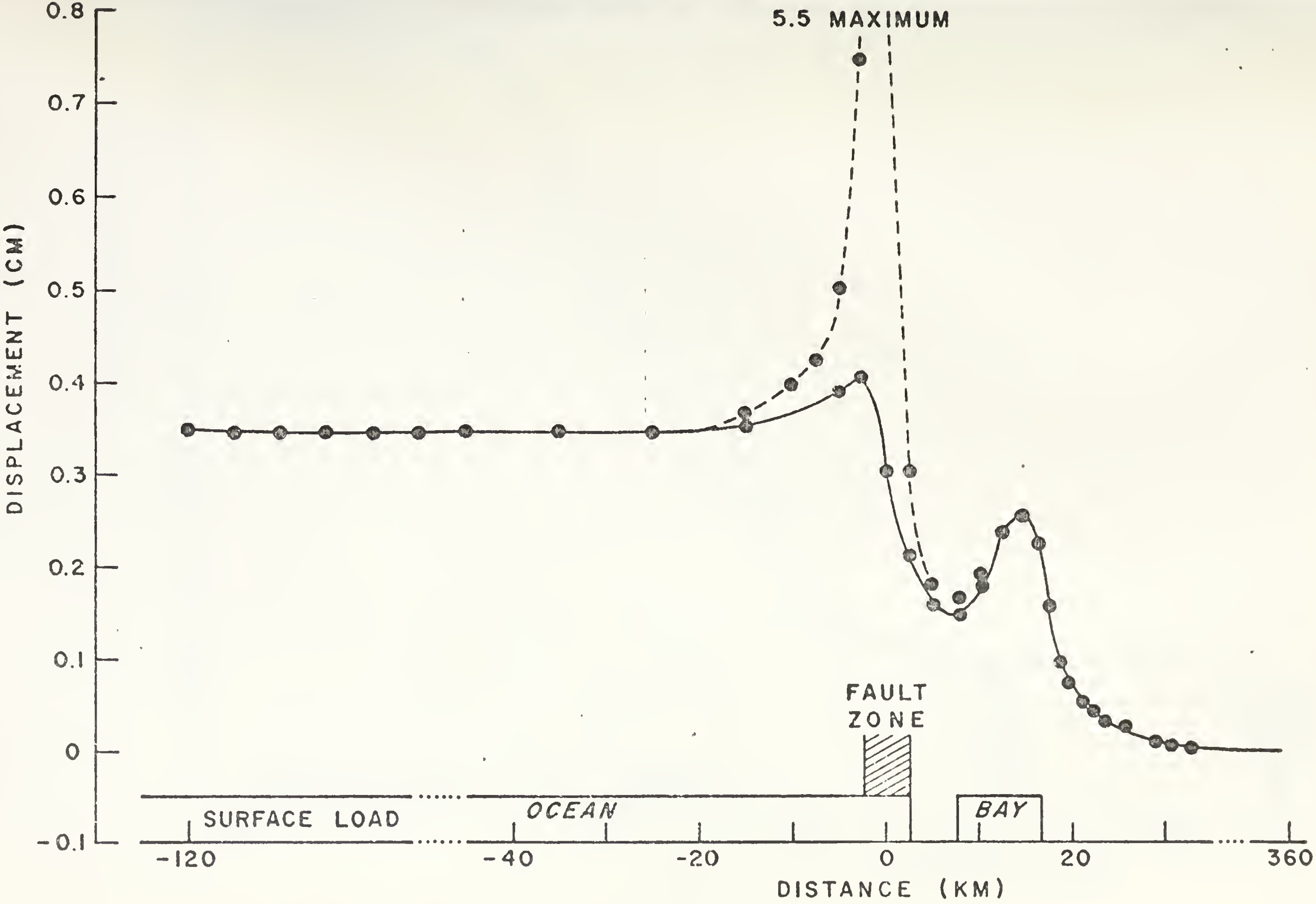


FIGURE 11

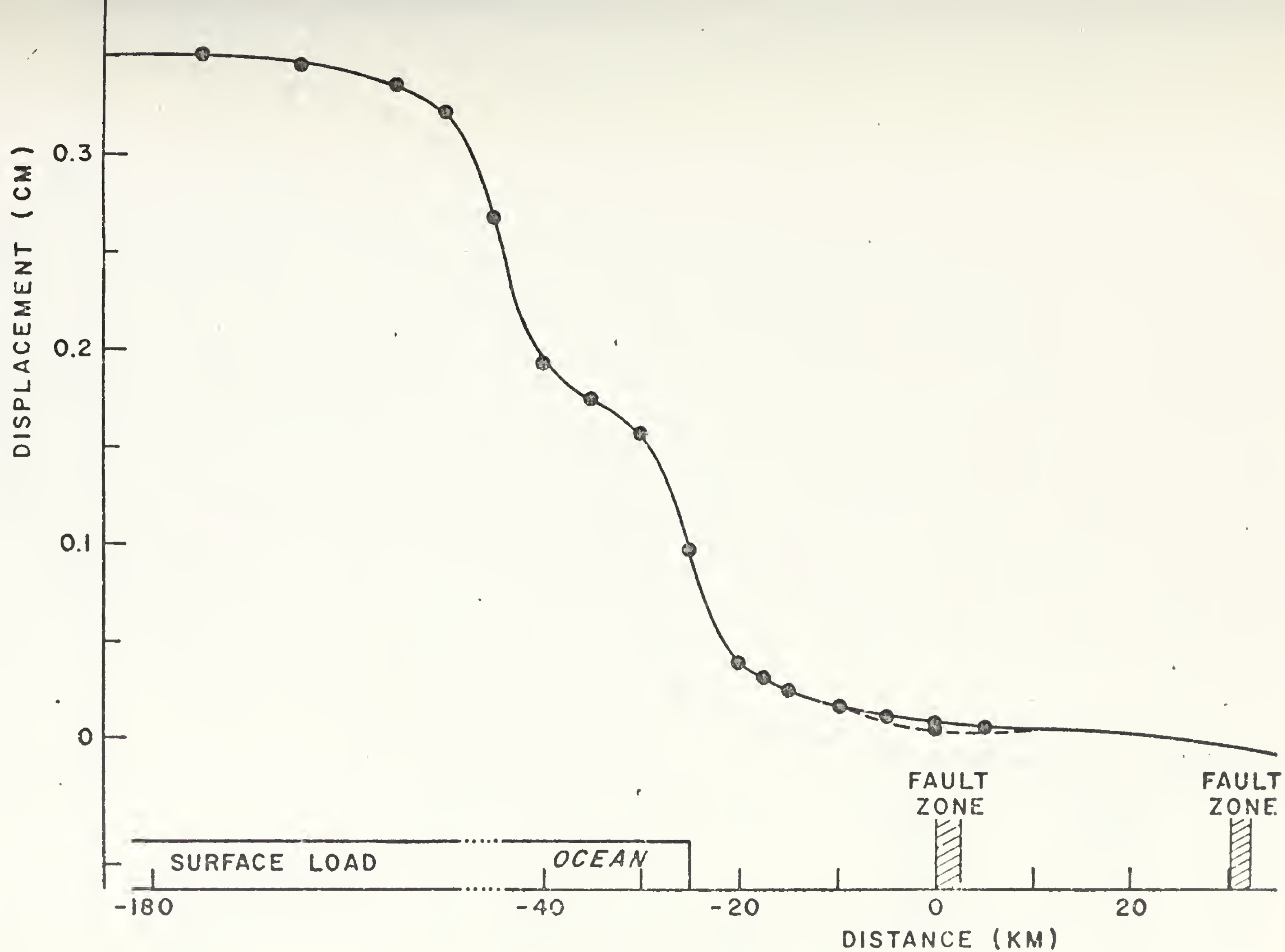

DISPLACEMENT (CM)
0.3
0.2
0.1
0
SURFACE LOAD
OCEAN
FAULT ZONE
FAULT ZONE
-180
-40
-20
0
20
DISTANCE (KM)

FIGURE 12

PROFILE C

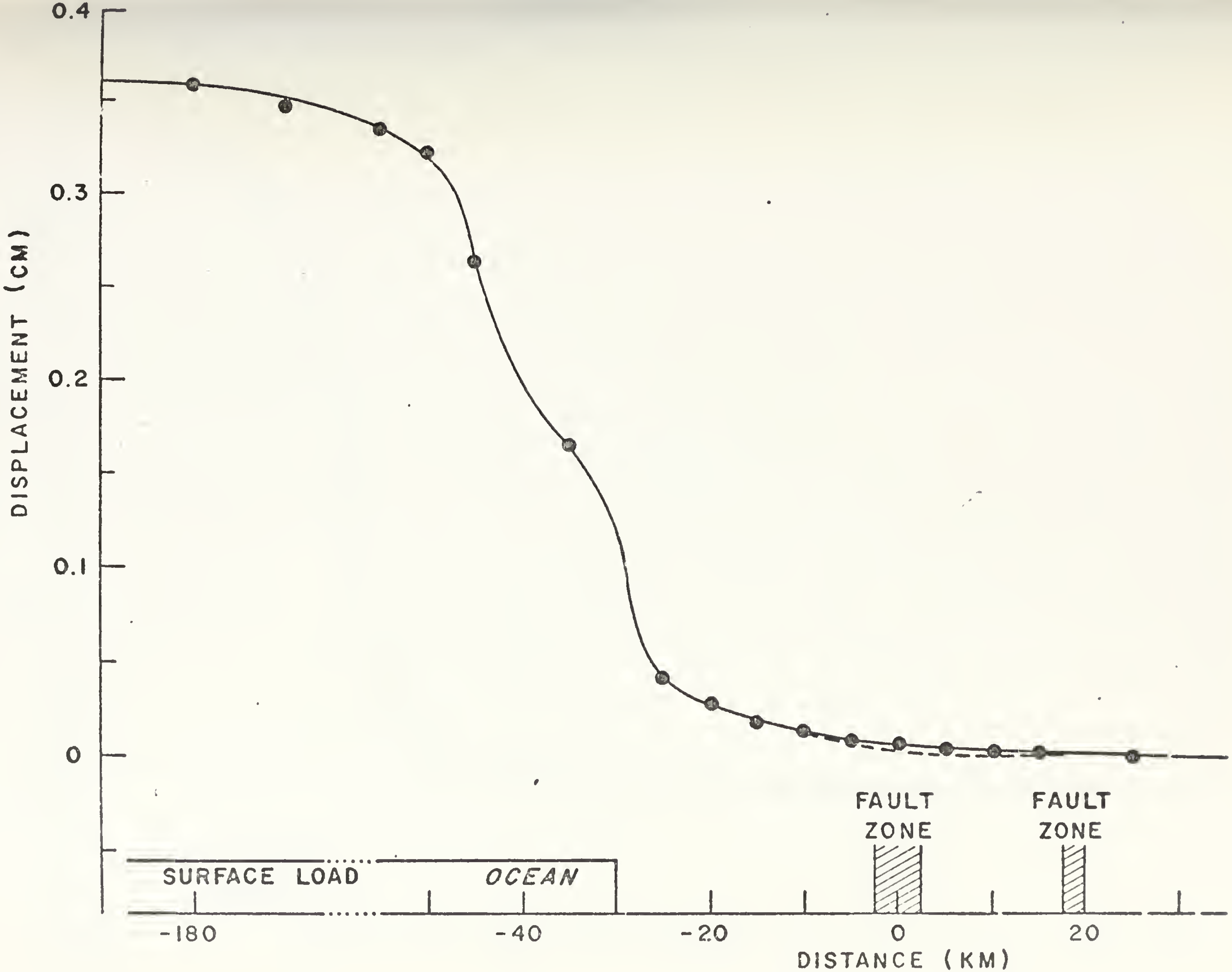


FIGURE 13

PROFILE D

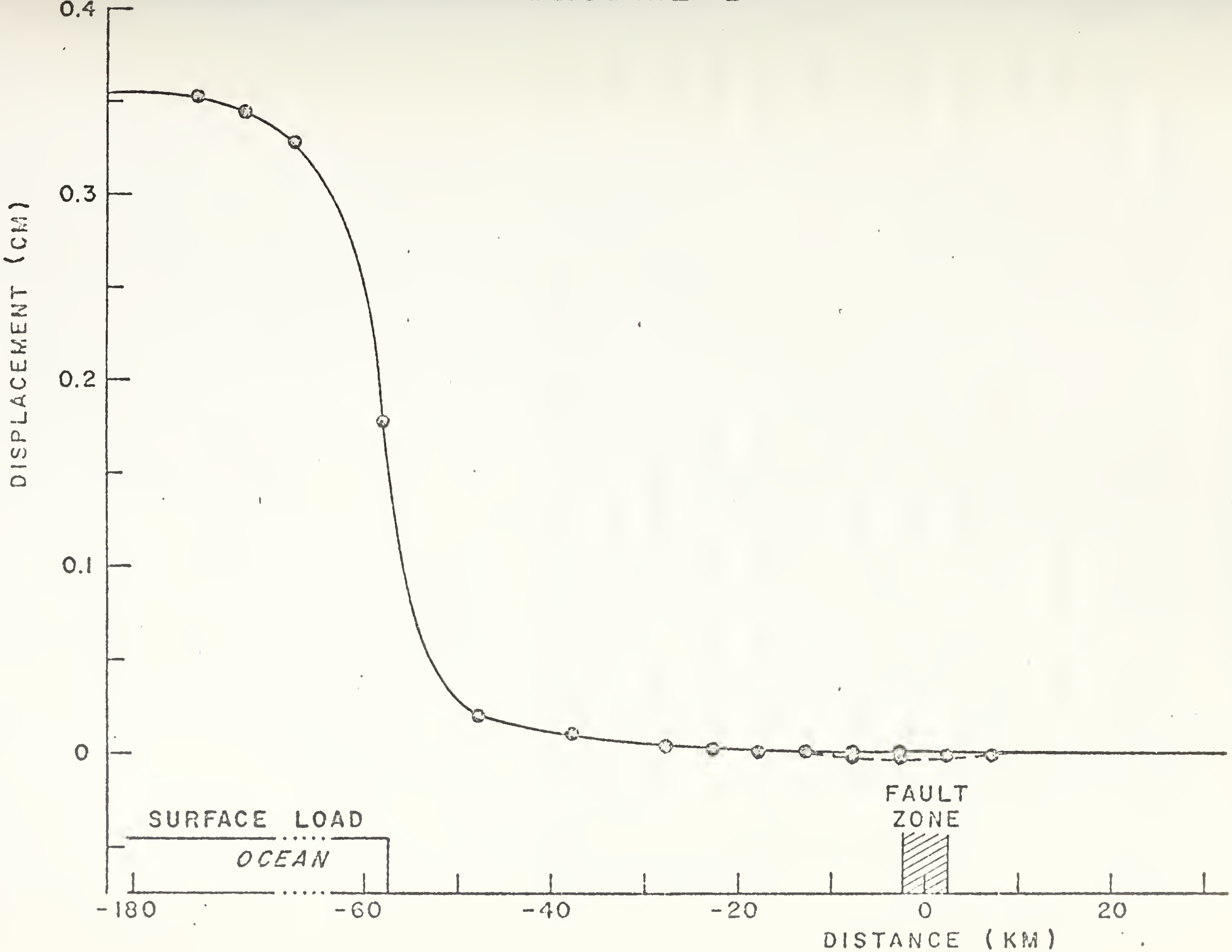


FIGURE 14

V. AREAS FOR FURTHER STUDY

Further study is definitely needed to develop the four California profiles into useable models. Modular construction of the grid is strongly encouraged when alterations are made to add further depth to the grid. Further work is needed to develop an effective and efficient three dimensional finite element computer program and model. This should provide a valuable tool for analyzing more detailed deformations resulting from a variety of loads applied to an inhomogeneous structure. This type of three dimensional finite element model could possibly be used in providing inputs to siloed missiles described in the introduction. Rainfall and barometric fluctuations could be used as the forcing functions to the finite element model instead of ocean tidal loading.

VI. CONCLUSIONS

The finite element model has demonstrated its usefulness in analyzing deformations resulting from a system of loads applied to an earth structure. If the structure is assumed to be of a homogeneous medium, then the finite element model produces results similar to that of Boussinesq models. In analyses of deformations in which the structure consists of an inhomogeneous medium the numerical method of finite elements has shown to be a most valuable asset. A logarithmic relationship between the magnitude of fault zone tilt and the distance of the fault zone from the edge of the load was shown to exist by manipulating the finite element model. Fault zone tilt was also shown to converge in magnitude for a given fault zone location when the fault zone shear modulus was decreased more than one order of magnitude from that of adjoining cells in the finite element grid.

APPENDIX A

DOCUMENTATION OF THE FINITE ELEMENT COMPUTER PROGRAM

The basic idea underlying the finite element concept is to substitute a simpler problem for the actual complex problem. If the simpler problem can be solved and if the resulting solution represents a feasible solution with acceptable accuracy, then the finite element technique has obviously served a useful purpose. The finite element method treats a continuum as an assembly of simple structural components or elements which are connected at a finite number of points, called notes.

The process of fitting a variety of geophysical measurements to the array of possible earth structures and their responses requires a finite element technique. The finite element computer program used in this paper was developed to meet the need for a simple mesh or grid construction allowing for analysis of deformations of axisymmetric and plane strain elastic structures. The program is based on the theory of finite elements as presented by O. C. Ziekiewicz and Y. K. Cheung [15]. Typical finite element programs are so tedious and time consuming in the design of the mesh and indexing of the nodes and cells that they tend to inhibit a thorough investigation of the variety of models that satisfy the geophysical measurements. The finite element program in this paper solves the problems of inversion, non-uniqueness,

and convergence in an efficient and effective manner without becoming overly complex. The program has facilities for processing in one run, several models which the investigator requires for validating his hypotheses.

The computer program is written in Fortran. This facilitates using the program on different systems. The program is composed of a main program and two subroutines, named ASAPS and QPLOT.

The main program constructs the grid, indexes and positions nodes and cells, assigns associated elastic parameters, sets up boundary conditions, and places the vertical surface force in position. The construction of the grid is accomplished by the standardization of five modular mesh types. The mesh types are described as coarse, intermediate, fine, left and right link connectors. The five mesh types are shown in figure 15.

The grid is the hub of the modeling phase. It is constructed by assembling the five mesh types or any combination of mesh types into the desired structure. The dimensions of the grid may vary depending on the number of mesh types used. The grid may be designed so that nodal points are closer spaced by using the fine mesh type in areas where deformations are expected to vary most rapidly. To provide the appropriate transition from a coarse mesh to either an intermediate or fine mesh requires the appropriate right or left link connectors to be inserted. Figure 16 illustrates a completed grid design. The type of grid design which has been discussed

SAMPLE GRID MESH

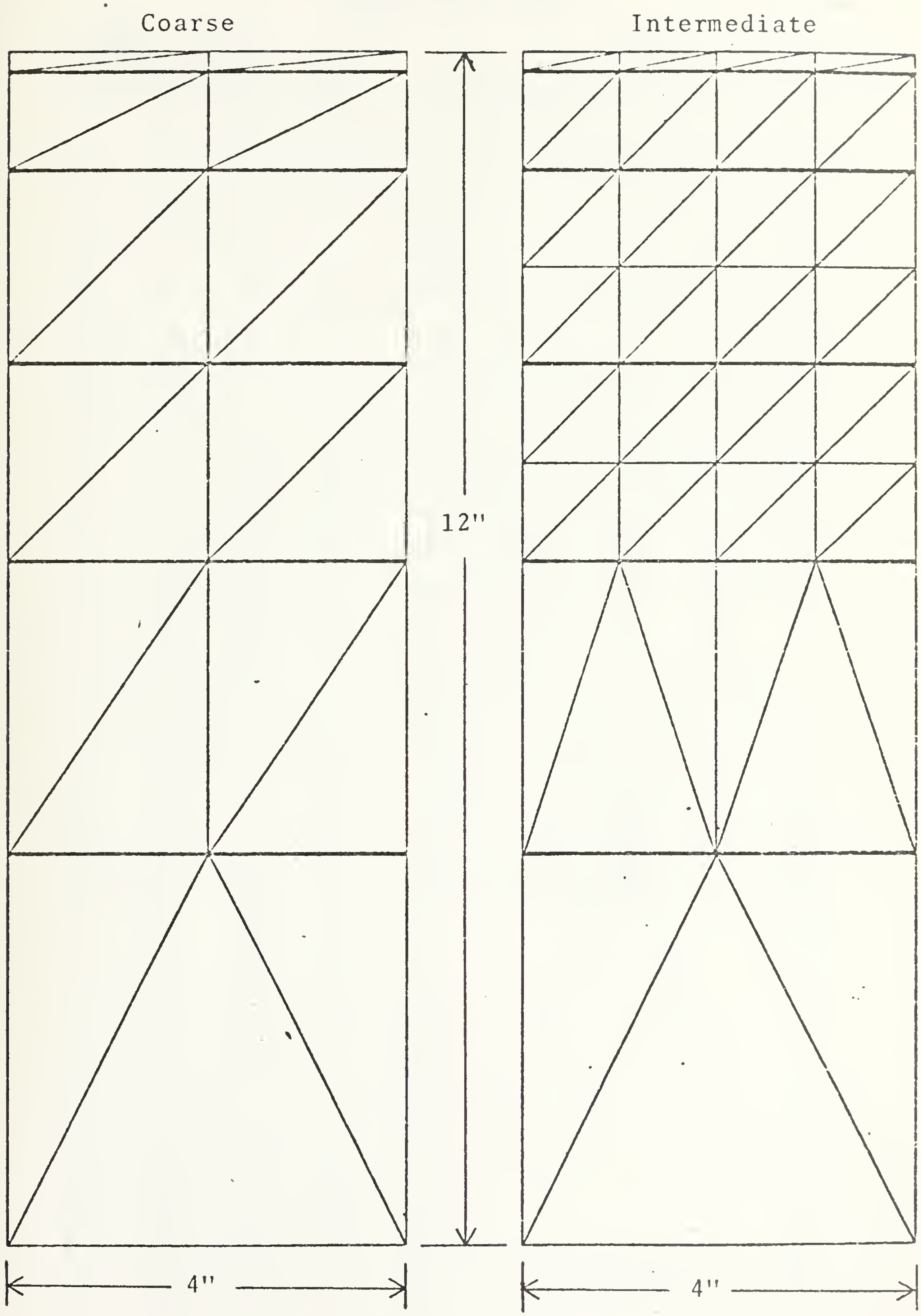


FIGURE 15A

SAMPLE GRID MESH

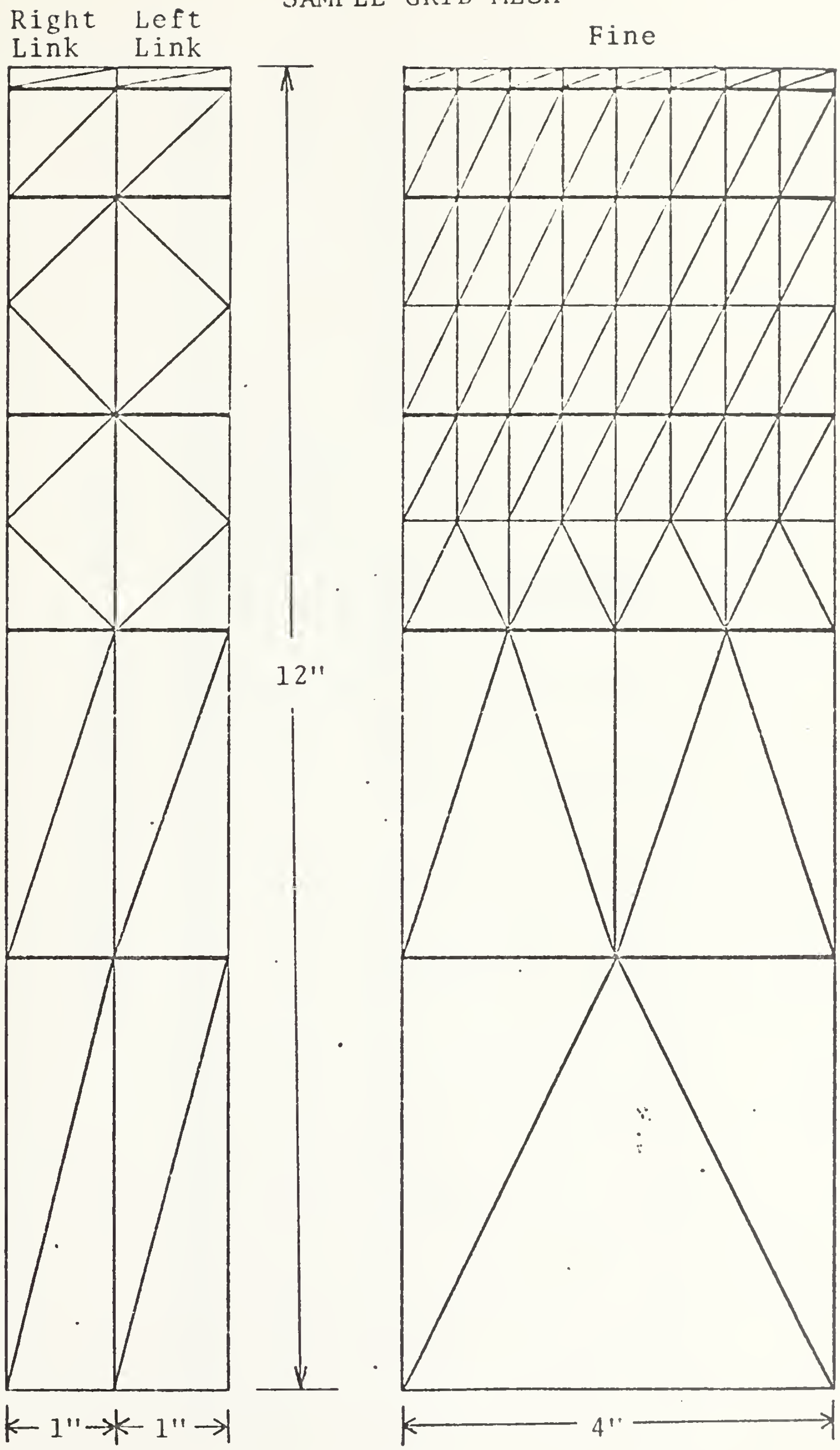
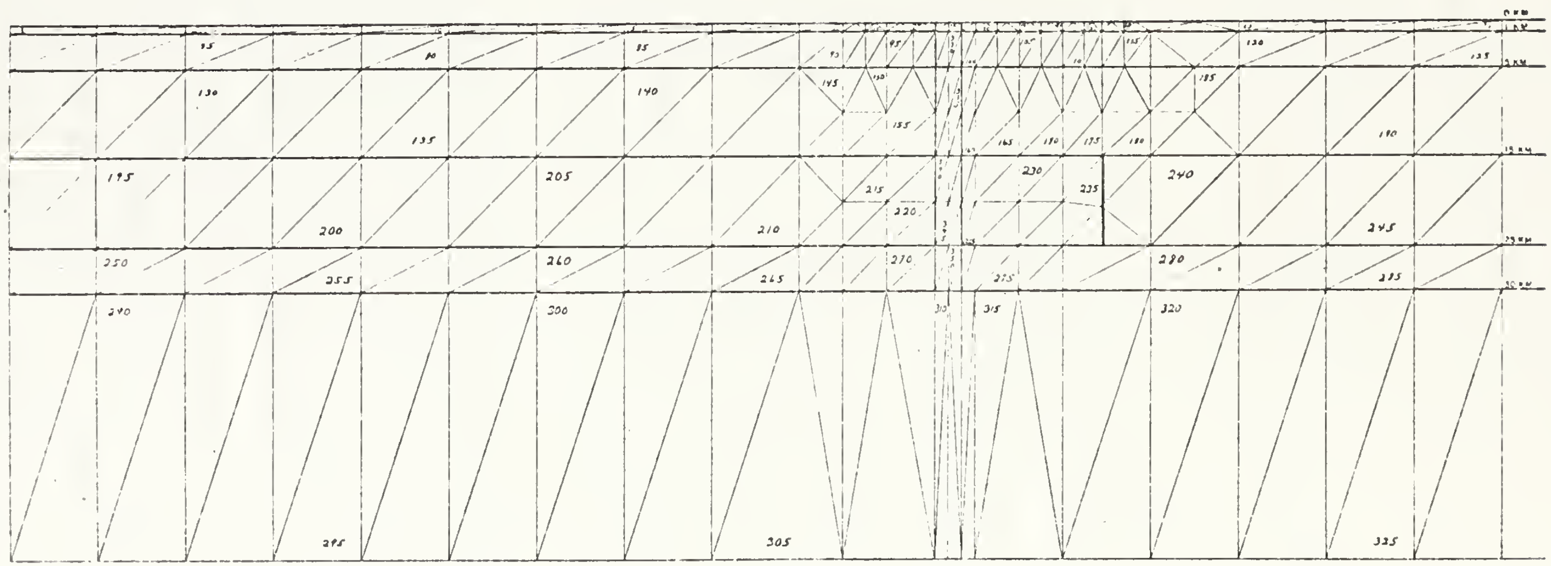


FIGURE 15B

SAMPLE OF COMPLETED GRID DESIGN



Scaling 0.2" = 1 km

1" = 5 km ══ NA = 5

FIGURE 16

above is only valid for plane strain modeling problems. Axisymmetric problems require a different type of grid which has ring-shaped elements with triangular cross sections. Thus for axisymmetric problems a manual grid construction is required and the automatic grid generator in the main program would be aborted. In any grid design the aspect ratio between the lengths of the two legs of the triangular element should not exceed 10:1.

An overlay procedure for elastic boundary conditions was devised so that structures composed of several materials are as easy to model as structures composed of a single material. Each elastic boundary condition is on a separate data card with the coordinates (in inches) which bound that particular set of elastic parameters. The overlay procedure is achieved by giving later specified boundary conditions precedence over former specified boundary conditions. This allows structures to be described by a basic structure which can be easily modified.

The ASAPS subroutine calculates the deformations of axisymmetric and plane-strain elastic structures, using the finite element technique [3]. This program employs the simple triangular element which is constrained to deform homogeneously (for axisymmetric problems the elements are ring-shaped with a triangular cross section). From the mechanical properties and dimensions of each element a stiffness matrix is formed for the entire assembly which relates nodal forces to nodal displacements. The x components and y components of nodal

displacements are the principal unknowns in the analysis and are determined from linear simultaneous equations of the form

$$F_t = \sum_{j=1}^{2N} K_{ij} D_j \qquad i = 1, 2, \ldots 2N$$

where N = number of nodes

F_i = components of force at the nodes; F_i and F_{N+i} are the horizontal and vertical components of force, respectively, at the i^{th} node

K_{ij} = stiffness matrix

D_j = nodal displacement components.

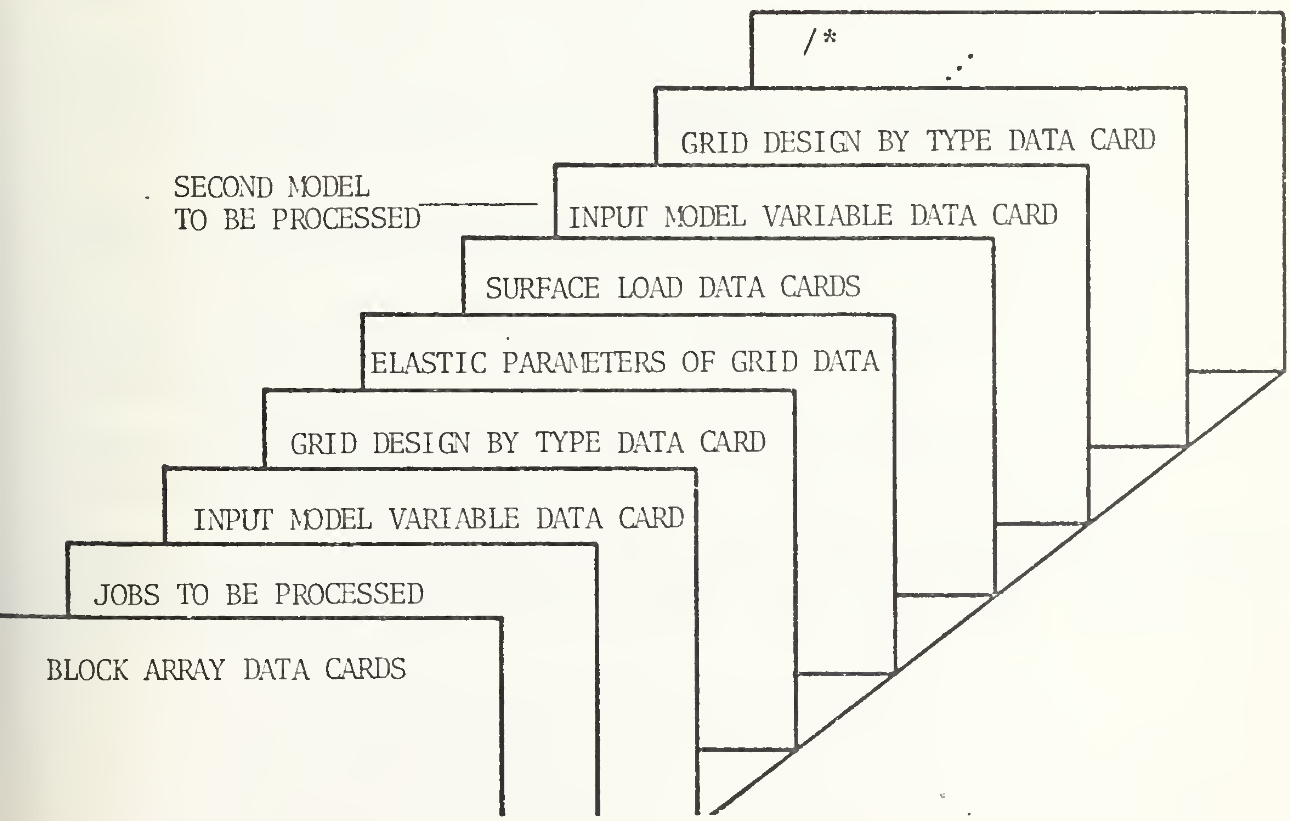
Young's overrelaxation method [5] was employed for solution of the equations. This is an iterative method used to improve the rate of convergence. The value of the overrelaxation parameter, W, must be specified with the input data. Because the optimum value of this parameter varies from problem to problem, efficient use of this method requires some experience. This inconvenience, however, is more than offset by the speed of this method and by the small amount of core which is required relative to simple and direct methods of solution. Other methods with comparable speed and storage place bothersome restrictions on the design and labeling of the finite element grid [3].

The subroutine QPLOT is a printer plot routine of the surface node displacements. It searches for the largest negative displacement and adds it to every nodal displacement. The plot then consists of all surface node displacements

plotted as a percent of the largest positive displacement. This was done to give the most clarity of detail possible to the plot. For each point on the plot, its associated surface node number and original displacements as calculated by subroutine ASAPS are printed.

Input Data Preparation

The first fifteen data cards of the data deck are called block array. They contain information about the construction of the grid from the five mesh types. These data cards are always required and always remain in the same position, regardless of the grid design. The following order of input data cards follows the block array data cards.



The above data cards in their proper formats are required to insure that the program will run successfully. The format for each parameter in the input data deck is described below.

Data Cards Supplied by Experimenter

Job Card

Columns	1 2
Format	I2
Parameter	JOB

Model Variables Data Card

Columns	1 4	5 9	10 14	15 19	20 24	25 29	30 34	35 39	40-44	45-46	47-48
Format	I4	I5	I5	I5	I5	I5	I5	F5.3	I5	I5	I5
Parameter	IPO	NBLKS	NZ	NF	KBND	IGEOM	ITERAT	W	NA	NB	NC

Grid Design by Mesh Type Data Card

Columns	1	2	3	4	.....	NBKS
Format	I1	I1	I1	I1		I 1
Parameter	NBLK(1)	NBLK(2)	NBLK(3)	NBLK(4)	.....	NBLK(BNLKS)

Elastic Parameters of Grid Data Cards
(There are NZ (Number) of These Data Cards)

Columns	1 5	6 10	11 15	16 20	21 27	28 34
Format	F5.1	F5.1	F5.1	F5.1	E7.1	E7.1
Parameter	YZ1	YZ2	YZ1	YZ2	UZ1	UZ2

Surface Loads Data Cards (There will be NF/10 (Rounded to Highest Whole Number) of these Data Cards)

Columns	1 5	6 10	11 15	16 20	21 25	26 30	31 35	36 40	41 45	46 50
Format	F5.1	F5.1	F5.1	F5.1	F5.1	F5.1	F5.1	F5.1	F5.1	F5.1
Parameter	FY	FY	FY	FY	FY	FY	FY	FY	FY	FY

The input parameters are described as follows:

JOB specifies the total number of models to be processed in one run.

IPO print option of which there are eight. The printout is divided into four parts:

A cell numbers, locations of associated nodes and elastic parameters.
B node numbers and location and associated values of NDISP, DISP.
C node number and location and associated displacement.
D diagram of vertical displacements of surface nodes.

IPO = 1 prints A, B, C, D
IPO = 2 prints B, C, D
IPO = 3 prints A, only surface nodes B, C, D
IPO = 4 prints only surface nodes B, C, D
IPO = 5 prints only data associated with surface nodes in B and C, D
IPO = 6 prints C, D
IPO = 7 prints only data associated with surface nodes in C, D
IPO = 8 prints D.

NBLKS total number of mesh types used in grid design

NZ total number of elastic boundary conditions incorporated in grid design.

NF total number of surface nodes in grid design.

KBND code for boundary conditions of the grid:

0 all sides free
1 bottom rigid
2 sides rigid
3 bottom and sides rigid.

IGEOM code for plain strain or axisymmetric type of problem:

1 plain strain problem and stress strain calculations in ASAPS will be bypassed.
2 axisymmetric problem and stress strain calculations will be accomplished in ASAPS

ITERAT number of iterations taken for solution of simultaneous equations.

W overrelaxation parameter used to reduce solution time and speed convergence. Value should be between 1.6 and 1.7.

NA exponent of ten used as scale factor for nodal coordinates.

NB exponent of ten used as scale factor for surface nodal forces.

NC exponent of ten used as scale factor for nodal displacement (zero if cm. is required and cgs units are used for NA and NB)

ND exponent of ten used as scale factor for elastic parameters (Lame's constants). If elastic parameters are loaded in exponental form, then ND = 0.

NBLK subscripted variable containing coded quantity: of the grid design by mesh type.

1 course
2 left link connector
3 intermediate
4 fine
5 right link connector

YZ1 Y coordinate of top left hand corner of boundary condition (in grid coordinates, i.e., inches)

YZ2 Y coordinate of bottom left hand corner of boundary condition (in grid coordinates, i.e., inches)

XZ1 X coordinate of top right hand corner of boundary condition (usually zero and also in grid coordinates, i.e., inches)

XZ2 X coordinate of top right hand corner of boundary condition (in grid coordinates, i.e., inches).

UZ1 Lamé constant Lamda (λ) coefficient of elasticity.

UZ2 Lamé constant Mu (μ) coefficient of elasticity.

FY(I) normal force at each of the surface nodes.

Example calculation for solving Lame's constants given a velocity structure:

Find λ and μ for the sample velocity structure in the zone where v_p = 6.0 km/sec assuming Poisson ratio of .25.

Poisson ratio $\sigma = \frac{\lambda}{2(\lambda+\mu)}$ (1)

Velocity for elastic waves (longitudinal waves, P waves) $v_p = \left(\frac{\lambda+2\mu}{\rho}\right)^{1/2}$ (2)

where ρ = density and is obtained from the Nafe-Drake curve which displays the variation of density with P wave velocity.

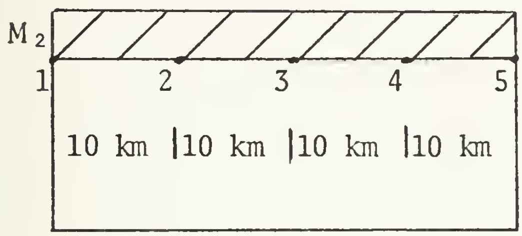
For v_p = 6.0 km/sec $\Longrightarrow$ $\rho = 2.75\ g/cm^3$

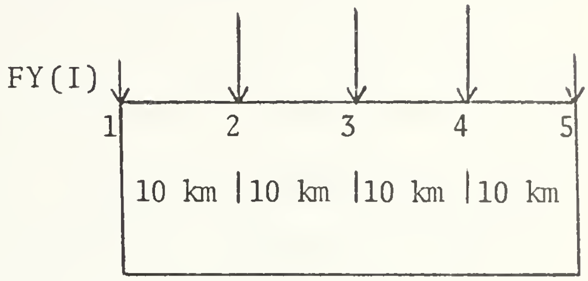
Now, solving equations 1 and 2 above simultaneously for λ and μ.

Equation I	Equation 2
$.25 = \frac{\lambda}{2(\lambda+\mu)}$	$6 \times 10^5 = \left(\frac{\lambda+2\mu}{2.75}\right)^{1/2}$
$.5 = \frac{\lambda}{\lambda+\mu}$	$6 \times 10^5 = \left(\frac{3\lambda}{2.75}\right)^{1/2}$
$.5\lambda + .5\mu = \lambda$	$\lambda = 3.3 \times 10^{11}$
$\lambda = \lambda$	$\mu = 3.3 \times 10^{11}$ dynes/cm^2
	ANS.

Example calculation for determining scale factor (NB) and normal surface forces (FY) given the amplitude of the M_2 frequency of third tidal spectrum.

Find FY(I), given M_2 = 50 centimeters?





Force = Pressure · Area

Pressure = $\rho g h$ = 1 g/cm^3 x 9.8 x 10^2 cm/sec^2 x 50 cm

= 4.9 x 10^4

where ρ = density of sea water

g = gravity

h = amplitude of M_2

Area = Distance between nodes · 1 cm

= 10 x 10^5 cm^2

(Note: 1 cm is assumption of unit width of surface load.)

Force = 4.9 x 10^4 x 10 x 10^5 = 49 x 10^9 dynes at each interior node.

NB = 9.

Nodes located at edges of load should be loaded with half as much force, to simulate a uniform load.

Derivation of NA for coordinate system in kilometers:

Let smallest mesh distance (.2 inches) equal 1 km.

Then .2 inches = 1 km = 1 x 10^5 cm

.5 inches = 2.5 km = 2.5 x 10^5 cm.

1 inch = 5 km = 5 x 10^5 cm

therefore NA = 5.

Core Requirements

Core requirements obviously depend upon the size of the grid (i.e., the total number of cells and nodes). Variables O, P, Q, XC, YC, AREN, ELAM, and EMU are subscripted in common with the total number of cells in the grid. Variables XF, YF, FX, FY, MID, LID, and LIDS are subscripted in common with the total number of nodes in the grid. Finally, variables A, AR, B, MAP, NDISP, and DISP are subscripted in common with twice the total number of nodes in the grid. A program with 744 cells and 419 nodes required 280 K bits of core on an IBM 360-67 computer. This was a very large grid spanning 74 modeling inches. For most grids of 500 cells or less, 200 K bits of core would be sufficient to run the program, on an IBM 360-67 computer. Of course, core requirements will differ from system to system. The above mentioned core requirements are meant to be a guide.

Time Requirements

The time required to run the finite element computer program is a function of the grid size, the number of iterations required to obtain a convergent solution and the amount of printout required. An IBM 360-67 computer required approximately fifteen minutes to process a 744 cell, 74 inch model grid using 1,000 iterations and a complete printout (IPO = 1). This time may be considerably reduced if only information about the surface displacements is required. Using IPO = 8 on the same grid and 1,000 iterations only eight minutes was

required to run the program. Again, if IPO = 8 is used on the same grid and only 300 iterations are required the program ran in four minutes. Of course, time requirements will differ depending on the system and the overhead required. The above mentioned time requirements are meant as a guide.

Internal Forces

Internal forces can be processed by this finite element computer program in conjunction with surface loads or instead of surface loads. However, each internal node force must be loaded manually into the program by inserting the appropriate instruction in the main program. The calculation of the size of the nodal force is the same as if it were a surface force. The sign of the force specified determines force direction. Positive force is a downward force and a negative force is an upward force. The internal force at node I should be loaded in the following format,

FY(I) = (±)(force size) * 10.** NB

The internal force statements should be inserted between statement numbers FET03590 and FET03600 in the main program. If only internal forces are desired, blank data cards should be used in the input section for surface loads in the data deck.

Cutout Procedure

A problem faced by the experimenter is determination of a number of iterations sufficient to be assured that the solution has converged. This must be specified in the input data ITERAT. A number which far exceeds the point of

convergence wastes computer time and money. Therefore, in subroutine ASAPS variable CUT is specified in statement number FET04700. It is now arbitrarily set at 0.000001. CUT is compared with the percentage difference that some surface node has displaced in 25 iterations. Thus CUT is a percentage beyond which the experimenter feels the solution has converged and wishes to terminate the procedure. The surface node used for the comparison should be located either under the load system or in a zone where a large amount of deformation is expected to take place. The node number is determined by variable MIS specified in statement number FET04680 in the ASAPS subroutine. However, if the experimenter knows how many iterations are required and specifies that number in the input data, he may desire to by-pass the cutout procedure. This is accomplished by changing statement number FET06860 in ASAPS

```
from 901 IF(PERC.LE.CUT) GO TO 902
to   901 IF(PERC.LE.CUT) GO TO 104.
```

Definitions of Non-Input Variables

DISP(I) displacement in x-direction of node I (cgs units)

DISP(I+NODE) displacement in y-direction of node I

FX(I) horizontal force at node I

NCELL total number of cells in grid

NDISP(I) = 1 displacement is set at 0.1, i.e., node I is rigid in x-direction

= 0 displacement is unspecified at node I in x-direction

NDISP(I+NODE) same as NDISP(I) except in y-direction

NODE total number of nodal points in grid

O(I) node number of uppermost left node in cell I

P(I) clockwise node number from O(I) in cell I

Q(I) clockwise node number from P(I) in cell I

XF(I) x coordinate of node I

YF(I) y coordinate of node I

AREN(I) area of cell I

IBL(I) the node number beginning layer I of the grid (fine and intermediate have 9 layers course has 7 layers)

IEL(I) the node number that ends layer I of the grid

CUT percentage change in the distance that some surface node (determined by MIS) moves in 25 iterations. (See cutout procedure)

APPENDIX B

LISTING OF FINITE ELEMENT TECHNIQUE COMPUTER PROGRAM

```
      COMMON O(750),P(750),Q(750),XC(750,3),YC(750,3),
     2XF(425),YF(425),FX(425),FY(425),NDISP(0850),DISP(0850),
     3 XZ1(025),XZ2(025),YZ1(025),YZ2(025),UZ1(025),UZ2(025),
     4A(850,16),AREN(750),AR(850),B(850),ELAM(750),EMU(750),
     5LIDS(425,7),LID(425,8,2),MAP(850,16),MID(425,8),
     6NA,NB,NC,ND,NODE,NCELL,IGEOM,ITERAT,W,NF,IPO
      INTEGER O,P,Q,ON,PN,QN,OTEMP,OE,PE,QE
      DIMENSION NBLK(50),BLK(5,3,10),LAY(10),IEL(10),IBL(10)
C
C     XF,YF,FX,FY,MID,LID,LIDS...............N TOTAL NUMBER NODES
C     O,P,Q,XC,YC,AREN,ELAM,EMU..............M TOTAL NUMBER CELS
C     A,AR,B,MAP,NDISP,DISP..................2N
C
C    CODE  TYPE    CELLS    NODES    L,R LNK
C    1   COARSE     23       20      7,7
C    2   L.LNK      14       16      7,7
C    3   INT        57       40      9,9
C    4   FINE      101       64      9,9
C    5   R.LNK      14       16      9,7
C
11    FORMAT(1X,'END DUE TO VARIABLE NBLKS EQUAL ZERO')
12    FORMAT(1X,'END DUE TO VARIABLE NZ EQUAL ZERO')
13    FORMAT(1X,'END DUE TO VARIABLE NF EQUAL ZERO')
  100 FORMAT(2I5,6F5.1)
  101 FORMAT(8F10.2)
  102 FORMAT(2I5)
  103 FORMAT(I4,6I5,F5.3,4I5)
  104 FORMAT(10F5.1)
  105 FORMAT(50I1)
  600 FORMAT(10X,7F10.2,2I5)
605   FORMAT(4F5.1,2E7.1)
606   FORMAT(I5,4F10.2,2E10.2,I5)
666   FORMAT(I2)
  707 FORMAT(10F5.1)
  709 FORMAT(I5,3E12.4,3(E12.4,I5))
720   FORMAT(1H1,50X,'BLK MATRIX (5,3,10) REMAINS CONSTANT',//)
721   FORMAT(10X,'DATA CARD',/,20X,'IPO  NBLKS NZ   NF  KBND IGEOM ITERA
     1T    W      NA    NB    NC     ND',/)
722   FORMAT(19X,I4,5I5,3X,I5,3X,F5.3,4I5,//)
723   FORMAT(1X,'AREA',6X,'XZ1',7X,'XZ2',7X,'YZ1',7X,'YZ2',7X,'UZ1',7X,
     1'UZ2',2X,'AREA',/)
724   FORMAT(1H1,3X,4HCELL,1X,'O X-COORD',3X,'O Y-COORD',3X,
     1'P X-COORD',3X,'P Y-COORD',3X,'Q X-COORD',3X,'Q Y-COORD',4X,1HO,4X
     2,1HP,4X,1HQ,6X,5HLAMDA,6X,2HMU,4X,4HCELL)
725   FORMAT(1H1,1X,4HNODE,3X,7HX-COORD,5X,7HY-COORD,3X,6HXFORCE,9X,
```

```
     16HYFORCE,4X,6HNDISPX,1X,5HDISPX,1X,6HNDISPY,1X,5HDISPY,4X,4HNODE,
     23X,'NOTE COORD IN CGS UNITS',//)
  903 FORMAT(I5,6E12.4,3I5,2E12.4,I5)
  904 FORMAT(I5,8F10.2)
  905 FORMAT (4I5,6F10.2)
 9033 FORMAT(1X,I5,6E12.4,3I5,2X,2E10.2,2X,I3)
9034  FORMAT(10X,/,'NOTE COORD IN MESH UNITS',//)
C     WRITE(6,720)
      DO 59 I=1,5
      DO 59 J=1,3
      READ(5,104) (BLK(I,J,K),K=1,10)
C     WRITE (6,104) (BLK(I,J,K),K=1,10)
   59 CONTINUE
C
C    INPUT CARD*******JOB=NUMBER OF MODELS TO BE PRECESSED
C                THIS CARD PRECEDES DATA FOR MODELS (FORMATI2)
C
      READ(5,666) JOB
      DO 1000 NNK=1,JOB,1
C
C     INPUT CARD
C     IPO=OUTPUT PRINT OPTION    PRINT OUT IS IN FOUR PARTS
C     A-CELLNUMBERS,LOCATION OF ASSOCIATED NODES,LAMDA,MU
C     B-NODES AND THEIR ASSOCIATED LOCATION ,NDISP,DISP
C     C-DISPLACEMENTS IN X AND Y DIRECTION OF NODES
C     D-PRINTER PLOT OF SURFACE NODES
C     IPO=1 PRINTS A,B,C,D
C     IPO=2 PRINTS B,C,D
C     IPO=3 PRINTS A,ONLY SURFACE NODES IN B,C,D
C     IPO=4 PRINTS   ONLY SURFACE NODES IN B,C,D
C     IPO=5 PRINTS ONLY DATA ASSOCIATED WITH SURFACE NODES IN B AND C,D
C     IPO=6 PRINTS  C,D
C     IPO=7 PRINTS  ONLY  DISPLACEMEMTS OF SURFACE NODE C,D
C     IPO=8 PRINTS ONLY D
C     NBLKS=TOTAL NUMBER OF MESH TYPES USED IN GRID CONSTRUCTION
C     NZ=NUMBER ELASTIC BOUNDARY FIELDS.
C     NF=NUMBER SURFACE NODES.
C     KBND=0 ALL SIDES FREE (1) BOTTOM FIXED  (2) SIDES FIXED  (3) BOTTO
C      AND SIDES FIXED
C     IGEOM=1 IF STRESS-STRAIN CALCULATIONS IN SUBROUTINE ASAPS ARE TO
C       BE BYPASSED.  OTHERWISE IGEOM CAN BE ANY OTHER INTEGER.
C     ITERAT =NUMBER ITERATIONS TAKEN FOR SOLUTION OF SIMULTANEOUS
C       EQUATIONS IN SUBROUTINE ASAPS.
C     NA,NB,NC,ND=EXPONENT OF TEN USED AS SCALE FACTORS
C       NA--XMXX,YMXX,XF,YF(DIMENSIONS OF MESH,COORDINATES OF NODES)
C       NB--FY (NORMAL FORCE)
C      NC SCALE FACTOR FOR DISPLACMENTS
C       ND--ELAM,EMU (ELASTIC PARAMETERS)
C     W OVERRELAXATION PARAMETER 1.6<W<1.7
C
```

```
      WRITE(6,721)
      READ(5,103) IPO,NBLKS,NZ,NF,KBND,IGEOM,ITERAT,W,NA,NB,NC,ND
      WRITE(6,722)IPO,NBLKS,NZ,NF,KBND,IGEOM,ITERAT,W,NA,NB,NC,ND
      IF (NBLKS.EQ.0) GO TO 1111
C
C     INPUT CARD
C     NBLK(I)=TYPE OF I(TH) SECTION OF MESH, READING FROM LEFT TO RIGHT.
C       IE., SEQUENCE OF COARSE, L.LNK,INT.,FINE,R.LNK IS READ IN AS 123
C
      READ(5,105) (NBLK (I),I=1,NBLKS)
      X=0.
      Y=0.
      K=0
      IBLK1=NBLK(1)
      XMX=0
      YMX=12.
      DO 50 KN=1,NBLKS
      I=NBLK (KN)
      Y=0
      X=XMX
      XX=BLK(I,1,10)
      XMX=XMX+XX
      IF(KN.EQ.1) X=0
      DO 51 J=1,9
      IC=BLK(I,3,J)
      IF(IC.EQ.0) GO TO 50
      XV=BLK(I,1,J)
      YV=BLK(I,2,J)
C     WRITE (6,905) KN,J,K,IC,X,Y,XV,YV,XMX,XX
      GO TO (501,502,503,504),IC
 1009 CONTINUE
      XVV=XV
      IF (IC.EQ.2.OR.IC.EQ.5) XVV=2.*XV
      IF(X+XVV-XMX) 511,513,513
  511 X=X+XVV
      GO TO (501,502,503,504),IC
  513 Y=Y+YV
      IF(Y-YMX) 52,50,50
   52 X=XMX-XX
      GO TO 51
  501 K=K+1
      XC(K,1)=X
      YC(K,1)=Y
      XC(K,2)=X+XV
      YC(K,2)=Y
      XC(K,3)=X
      YC(K,3)=Y+YV
      K=K+1
      XC(K,1)=X+XV
      YC(K,1)=Y
```

```
      XC(K,2)=X+XV
      YC(K,2)=Y+YV
      XC(K,3)=X
      YC(K,3)=Y+YV
      GO TO 1009
  502 CONTINUE
      K=K+1
      XC(K,1)=X
      YC(K,1)=Y
      XC(K,2)=X+XV
      YC(K,2)=Y
      XC(K,3)=X
      YC(K,3)=Y+YV
      K=K+1
      XC(K,1)=X+XV
      YC(K,1)=Y
      XC(K,2)=X+2*XV
      YC(K,2)=Y+YV
      XC(K,3)=X
      YC(K,3)=Y+YV
      K=K+1
      XC(K,1)=X+XV
      YC(K,1)=Y
      XC(K,2)=X+2*XV
      YC(K,2)=Y
      XC(K,3)=X+2*XV
      YC(K,3)=Y+YV
      GO TO 1009
  503 CONTINUE
      K=K+1
      XC(K,1)=X
      YC(K,1)=Y
      XC(K,2)=X+XV
      YC(K,2)=Y
      XC(K,3)=X+XV
      YC(K,3)=Y+YV/2.
      K=K+1
      XC(K,1)=X
      YC(K,1)=Y
      XC(K,2)=X+XV
      YC(K,2)=Y+YV/2
      XC(K,3)=X
      YC(K,3)=Y+YV
      K=K+1
      XC(K,1)=X+XV
      YC(K,1)=Y+YV/2.
      XC(K,2)=X+XV
      YC(K,2)=Y+YV
      XC(K,3)=X
      YC(K,3)=Y+YV
```

```
      GO TO 1009                                                        FET01960
  504 CONTINUE                                                          FET01970
      K=K+1                                                             FET01980
      XC(K,1)=X                                                         FET01990
      YC(K,1)=Y                                                         FET02000
      XC(K,2)=X+XV                                                      FET02010
      YC(K,2)=Y                                                         FET02020
      XC(K,3)=X                                                         FET02030
      YC(K,3)=Y+YV/2.                                                   FET02040
      K=K+1                                                             FET02050
      XC(K,1)=X+XV                                                      FET02060
      YC(K,1)=Y                                                         FET02070
      XC(K,2)=X+XV                                                      FET02080
      YC(K,2)=Y+YV                                                      FET02090
      XC(K,3)=X                                                         FET02100
      YC(K,3)=Y+YV/2.                                                   FET02110
      K=K+1                                                             FET02120
      XC(K,1)=X                                                         FET02130
      YC(K,1)=Y+YV/2.                                                   FET02140
      XC(K,2)=X+XV                                                      FET02150
      YC(K,2)=Y+YV                                                      FET02160
      XC(K,3)=X                                                         FET02170
      YC(K,3)=Y+YV                                                      FET02180
      GO TO 1009                                                        FET02190
   51 CONTINUE                                                          FET02200
   50 CONTINUE                                                          FET02210
C                                                                       FET02220
C        FIND  AND REPLACE (REDUCE) ALL COMMON NODES TO LOWEST  ORDER   FET02230
C                                                                       FET02240
      J1=0                                                              FET02250
      K7=0                                                              FET02260
      DY=0.2                                                            FET02270
      DX=0.5                                                            FET02280
      K6=0                                                              FET02290
      X=0                                                               FET02300
      Y=0                                                               FET02310
      YMX=12.2                                                          FET02320
      XMX=0                                                             FET02330
      DO 969 N=1,NBLKS                                                  FET02340
      KN=NBLK(N)                                                        FET02350
  969 XMX= XMX+ BLK(KN,1,10)                                            FET02360
      XMXX=(5.*XMX)*10.**NA                                             FET02370
      YMXX=(5.*YMX)*10.**NA                                             FET02380
      NYI = 2 + YMX                                                     FET02390
      NXI=1+XMX/DX                                                      FET02400
      DO 950 I1=1,NYI                                                   FET02410
      DO 951 I2=1,NXI                                                   FET02420
      K7=0                                                              FET02430
      IH=0                                                              FET02440
      DO 953 I=1,K                                                      FET02450
```

```
      DO 955 L=1,3
      IF(XC(I,L).GE.X.AND.XC(I,L).LT.X+DX.AND.
     C    YC(I,L).GE.Y.AND.YC(I,L).LT.Y+DY)GO TO 952
      GO TO 955
  952 K7=K7+1
      IF(IH.EQ.0)J1=J1+1
      JT=J1
      IH=1
      GO TO (1,2,3),L
    1 O(I)=JT
      GO TO 954
    2 P(I)=JT
      GO TO 954
    3 Q(I)=JT
  954 CONTINUE
      GO TO 953
  955 CONTINUE
  953 CONTINUE
      X=X+DX
  951 CONTINUE
      X=0
      Y=Y+DY
      DY=1
  950 CONTINUE
  959 CONTINUE
C     DO 909 J=1,K
C     WRITE(6,903) J,XC(J,1),YC(J,1),XC(J,2),YC(J,2),XC(J,3),YC(J,3),
C    2O(J),P(J),Q(J)
C 909 CONTINUE
      NCELL=K
      IF(O(K).GE.P(K).AND.O(K).GE.Q(K)) NODE=O(K)
      IF(P(K).GE.O(K).AND.P(K).GE.Q(K)) NODE=P(K)
      IF(Q(K).GE.O(K).AND.Q(K).GE.P(K)) NODE=Q(K)
      DO 703 J=1,K
      I=O(J)
        XF(I)=(5.*XC(J,1))*10.**NA
        YF(I)=(5.*YC(J,1))*10.**NA
      I=P(J)
        XF(I)=(5.*XC(J,2))*10.**NA
        YF(I)= (5.*YC(J,2))*10.**NA
      I=Q(J)
        XF(I)=(5.*XC(J,3))*10.**NA
        YF(I)=(5.*YC(J,3))*10.**NA
  703 CONTINUE
      IF (NZ.EQ.0) GO TO 1112
      WRITE(6,723)
      DO 20 N=1,NZ
C
C     INPUT CARDS
C     ELASTIC BOUNDARY FIELDS MAY OVERLAP I.E. ARE INDEPENDENT OF MESH
```

```
C       LATTER BOUNDARY CONTIONS TAKE PRECEDENCE OVER FORMER BOUNDARIES
C
C       NOTE REVERSE READ Y1,Y2 THEN X1,X2...ETC IN READ(5,605)
C       XZ1,YZ1=X,Y COORDINATES OF TOP LEFT-HAND CORNER OF BOUNDARY
C       XZ2=X COORDINATE OF RIGHT-HAND BOUNDARY.
C       YZ2=Y COORDINATE OF BOTTOM BOUNDARY.
C       UZ1,UZ2=LAMBDA,MU,RESPECTIVELY.
C
        READ(5,605) YZ1(N),YZ2(N),XZ1(N),XZ2(N),UZ1(N),UZ2(N)
        WRITE(6,606)N,XZ1(N),XZ2(N),YZ1(N),YZ2(N),UZ1(N),UZ2(N),N
   20   CONTINUE
        JJ=1
        KCC=0
        DO 112 N=1,NBLKS,1
        KN=NBLK(N)
        GO TO(106,107,108,109,107),KN
106     KC=23
        GO TO 110
107     KC=14
        GO TO 110
108     KC=57
        GO TO 110
109     KC=101
110     KCC=KCC+KC
        DO 801 I=1,NZ,1
        DO 111 J=JJ,KCC,1
        E=XZ1(I)
        F=XZ2(I)
        G=YZ1(I)
        H=YZ2(I)
        IF(XC(J,1).LT.E.OR.XC(J,2).LT.E.OR.XC(J,3).LT.E.OR.
      2   XC(J,1).GT.F.OR.XC(J,2).GT.F.OR.XC(J,3).GT.F) GO TO 111
        IF(YC(J,1).LT.G.OR.YC(J,2).LT.G.OR.YC(J,3).LT.G.OR.
      2   YC(J,1).GT.H.OR.YC(J,2).GT.H.OR.YC(J,3).GT.H) GO TO 111
        ELAM(J)=UZ1(I)*10.**ND
        EMU(J)=UZ2(I)*10.**ND
111     CONTINUE
801     CONTINUE
        JJ=JJ+KC
112     CONTINUE
        IF(IPO.GE.3)GO TO 113
        WRITE(6,724)
        WRITE(6,9034)
        DO 113 J=1,K,1
  907   WRITE(6,9033) J,XC(J,1),YC(J,1),XC(J,2),YC(J,2),XC(J,3),YC(J,3),
      2D(J),P(J),Q(J),ELAM(J),EMU(J),J
113     CONTINUE
          IF (NF.EQ.0) GO TO 1113
C
C       INPUT CARDS
```

```
C       FY=NORMAL FORCE.                                                    FET03460
C                                                                           FET03470
        READ(5,707) (FY(I),I=1,NF)                                          FET03480
        DO 705 I=1,NODE                                                     FET03490
        FX(I)=0                                                             FET03500
        IF(I+NF.GT.NODE) GO TO 5555                                         FET03510
        FY(I+NF) = 0                                                        FET03520
 5555   CONTINUE                                                            FET03530
        FY(I)=FY(I)*10.**NB                                                 FET03540
        NDISP(I)=0                                                          FET03550
        NDISP(I+NODE)=0                                                     FET03560
         DISP(I)=1                                                          FET03570
        DISP(I+NODE)=1                                                      FET03580
  705   CONTINUE                                                            FET03590
        IF(KBND.EQ.0)GO TO 4445                                             FET03600
        DO 60 J=1,9,1                                                       FET03610
        NBL=1                                                               FET03620
        IF(J.EQ.4.OR.J.EQ.6.OR.J.GE.8) GO TO 61                             FET03630
        IF(J.EQ.7) NBL=2                                                    FET03640
        LAY(J)=1                                                            FET03650
        DO 62 N=1,NBLKS,1                                                   FET03660
        K=NBLK(N)                                                           FET03670
        GO TO(63,64,65,66,64,66),K                                          FET03680
 63     LAY(J)=LAY(J)+2                                                     FET03690
        GO TO 62                                                            FET03700
 64     LAY(J)=LAY(J)+1                                                     FET03710
        GO TO 62                                                            FET03720
 65     LAY(J)=LAY(J)+4                                                     FET03730
        GO TO 62                                                            FET03740
 66     LAY(J)=LAY(J)+8/NBL                                                 FET03750
 62     CONTINUE                                                            FET03760
        GO TO 60                                                            FET03770
 61     IF(J.GE.8)GO TO 67                                                  FET03780
        LAY(J)=0                                                            FET03790
        DO 68 N=1,NBLKS,1                                                   FET03800
        K=NBLK(N)                                                           FET03810
        GO TO (68,69,70,71,68,71),K                                         FET03820
 69     LAY(J)=LAY(J)+1                                                     FET03830
        GO TO 68                                                            FET03840
 70     LAY(J)=LAY(J)+4                                                     FET03850
        GO TO 68                                                            FET03860
 71     LAY(J)=LAY(J)+8                                                     FET03870
 68     CONTINUE                                                            FET03880
        GO TO 60                                                            FET03890
 67     IF(J.EQ.9)GO TO 72                                                  FET03900
        LAY(J)=1                                                            FET03910
        DO 73 N=1,NBLKS,1                                                   FET03920
        K=NBLK(N)                                                           FET03930
        GO TO (74,75,74,74,75,74),K                                         FET03940
 74     LAY(J)=LAY(J)+2                                                     FET03950
```

```
      GO TO 73                                                          FET03960
75    LAY(J)=LAY(J)+1                                                   FET03970
73    CONTINUE                                                          FET03980
      GO TO 60                                                          FET03990
72    LAY(J)=NBLKS+1                                                    FET04000
60    CONTINUE                                                          FET04010
      IBL(1)=1                                                          FET04020
      IEL(1)=LAY(1)                                                     FET04030
      DO 76 J=2,9,1                                                     FET04040
      IBL(J)=IEL(J-1)+1                                                 FET04050
      IEL(J)=IEL(J-1)+LAY(J)                                            FET04060
76    CONTINUE                                                          FET04070
      IF(KBND.EQ.2) GO TO 4446                                          FET04080
      KBL=IBL(9)                                                        FET04090
      KEL=IEL(9)                                                        FET04100
      DO 4444 I=KBL,KEL,1                                               FET04110
      NDISP(I)=1                                                        FET04120
      NDISP(I+NODE)=1                                                   FET04130
4444  CONTINUE                                                          FET04140
      IF(KBND.EQ.1) GO TO 4445                                          FET04150
4446  DO 4445 I=1,8,1                                                   FET04160
      IF(I.EQ.4.OR.I.EQ.6)GO TO 4445                                    FET04170
      NDISP(IBL(I))=1                                                   FET04180
      NDISP(IEL(I))=1                                                   FET04190
      NDISP(IBL(I)+NODE)=1                                              FET04200
      NDISP(IEL(I)+NODE)=1                                              FET04210
4445  CONTINUE                                                          FET04220
      NOD=0                                                             FET04230
      IF(IPO.GE.6) GO TO 440                                            FET04240
      IF(IPO.EQ.1.OR.IPO.EQ.2)NOD=NODE                                  FET04250
      IF(IPO.GE.3.AND.IPO.LE.5)NOD=NF                                   FET04260
      WRITE(6,725)                                                      FET04270
80    DO 708 I=1,NOD,1                                                  FET04280
      WRITE(6,709) I,XF(I),YF(I),FX(I),FY(I),NDISP(I),DISP(I),          FET04290
     2NDISP(I+NODE),DISP(I+NODE),I                                      FET04300
440   CONTINUE                                                          FET04310
  708 CONTINUE                                                          FET04320
      CALL ASAPS(JOB)                                                   FET04330
 1000 CONTINUE                                                          FET04340
      GO TO 14                                                          FET04350
1111  WRITE(6,11)                                                       FET04360
      GO TO 14                                                          FET04370
1112  WRITE(6,12)                                                       FET04380
      GO TO 14                                                          FET04390
1113  WRITE(6,13)                                                       FET04400
14    STOP                                                              FET04410
      END                                                               FET04420
```

```
      SUBROUTINE ASAPS (NJ)                                              FET04440
C                                                                        FET04450
C                  * FINITE ELEMENT METHOD FOR AXISYMMETRIC*             FET04460
C                  *     AND PLANE-STRAIN PROBLEMS         *             FET04470
C                  *****************************************             FET04480
C                                                                        FET04490
      COMMON O(750),P(750),Q(750),XC(750,3),YC(750,3),                   FET04500
     2X (425),Y (425),FX(425),FY(425),NDISP(0850),DISP(0850),            FET04510
     3 XZ1(025),XZ2(025),YZ1(025),YZ2(025),UZ1(025),UZ2(025),            FET04520
     4A(850,16),AREN(750),AR(850),B(850),ELAM(750),EMU(750),             FET04530
     5LIDS(425,7),LID(425,8,2),MAP(850,16),MID(425,8),                   FET04540
     6NA,NB,NC,ND,NODE,NCELL,IGEOM,ITERAT,W,NF,IPO                       FET04550
      DIMENSION XPLOT(100),DIST(5)                                       FET04560
      INTEGER O,P,Q,ON,PN,QN,OTEMP,OE,PE,QE                              FET04570
   13 FORMAT(6X,I3,2F10.5)                                               FET04580
  110 FORMAT(18X,13HDISPLACEMENTS)                                       FET04590
111   FORMAT(4X,4HNODE,7X,6HX-COMP,9X,6HY-COMP,9X,7HX-COORD,8X,          FET04600
     17HY-COORD,/)                                                       FET04610
  113  FORMAT(5X,I3,4X,E11.4,4X,E11.4,2E12.4)                            FET04620
  163 FORMAT(1H1,1X,' ITERATION',I4)                                     FET04630
  830 FORMAT(5X,69HBLOW UP OF XUN IN SOLUTION OF EQUATIONS ...POSSIBLE EFET04640
     1RROR IN X Y DATA)                                                  FET04650
  831 FORMAT(5X,19HNODE  X(I)     Y(I))                                  FET04660
      LINES=1                                                            FET04670
      MIS=NF/6                                                           FET04680
      MIDD=MIS+NODE                                                      FET04690
      CUT=0.000001                                                       FET04700
      LPLOT=NF                                                           FET04710
      FILL=100.                                                          FET04720
      IIHR=0                                                             FET04730
      NPT=IPO                                                            FET04740
      RNG=0.                                                             FET04750
      DMIN=0.                                                            FET04760
      DAY1=1.                                                            FET04770
      IT=2*NODE                                                          FET04780
      DO 51 I=1,NODE,1                                                   FET04790
      DO 52 J=1,7,1                                                      FET04800
      LIDS(I,J)=0                                                        FET04810
   52 CONTINUE                                                           FET04820
      DO 53 J=1,8,1                                                      FET04830
      MID(I,J)=0                                                         FET04840
      DO 54 K=1,2,1                                                      FET04850
      LID(I,J,K)=0                                                       FET04860
   54 CONTINUE                                                           FET04870
   53 CONTINUE                                                           FET04880
   51 CONTINUE                                                           FET04890
C        MID(I,J)=NATUALLY OCCURING NODE PAIRS...MID(I,K)=J  I,J=NODES   FET04900
C                                                              K=1-8     FET04910
C                                                                        FET04920
C       LIDS(I,J)=ALL CELLS WITH NODE I....J=1-7                         FET04930
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C                                                                         FET04940
C        LID(I,J,K)=CELLS WITH NODE PAIR I,L...L=MID(I,J),K=1-2           FET04950
      DO 60 I=1,NODE,1                                                    FET04960
      MID(I,1)=I                                                          FET04970
   60 CONTINUE                                                            FET04980
      DO 61 I=1,NCELL,1                                                   FET04990
      ON=O(I)                                                             FET05000
      PN=P(I)                                                             FET05010
      QN=Q(I)                                                             FET05020
      DO 62 IC=1,3,1                                                      FET05030
      IF(IC.EQ.1) JT=ON                                                   FET05040
      IF(IC.EQ.2) JT=PN                                                   FET05050
      IF(IC.EQ.3) JT=QN                                                   FET05060
      DO 55 J=1,7,1                                                       FET05070
  999 IF(LIDS(JT ,J).EQ.0) GO TO 63                                       FET05080
   55 CONTINUE                                                            FET05090
      WRITE(6,833)JT                                                      FET05100
      WRITE(6,834)                                                        FET05110
      WRITE(6,835)(LIDS(JT,IE),IE=1,7,1),I                                FET05120
      STOP                                                                FET05130
  833 FORMAT(3X,39HERROR...TOO MANY CELLS ADJACENT TO NODE,I3)            FET05140
  834 FORMAT(3X,26HCELLS ADJACENT TO THE NODE)                            FET05150
  835 FORMAT(3X,8I6)                                                      FET05160
   63 LIDS(JT,J)=I                                                        FET05170
      J=J+1                                                               FET05180
   62 CONTINUE                                                            FET05190
      DO 64 ITP=1,3,1                                                     FET05200
      DO 73 J=2,8,1                                                       FET05210
      IF(MID(ON ,J).EQ.0) GO TO 66                                        FET05220
      IF(MID(ON ,J).EQ.PN    ) GO TO 67                                   FET05230
   73 CONTINUE                                                            FET05240
   66 MID(ON,J)=PN                                                        FET05250
      J=J+1                                                               FET05260
   67 DO 74 J=2,8,1                                                       FET05270
      IF(MID(ON ,J).EQ.0) GO TO 68                                        FET05280
      IF(MID(ON ,J).EQ.QN    ) GO TO 69                                   FET05290
   74 CONTINUE                                                            FET05300
      IF(J .EQ.9) WRITE(6,850) ON,(MID(ON,K),K=2,8,1),QN                  FET05310
      IF(J.EQ.9) STOP                                                     FET05320
  850 FORMAT(2X,20HERROR IN DATA, NODE ,I3,2X,20HADJACENT TO NODES    ,8IFET05330
     15)                                                                  FET05340
   68 MID(ON,J)=QN                                                        FET05350
      J=J+1                                                               FET05360
   69 OTEMP=ON                                                            FET05370
      ON=PN                                                               FET05380
      PN=QN                                                               FET05390
      QN=OTEMP                                                            FET05400
   64 CONTINUE                                                            FET05410
   61 CONTINUE                                                            FET05420
      DO 70 I=1,NODE,1                                                    FET05430
```

```
      DO 71  J=2,8,1                                                    FET05440
      LTEMP=MID(I,J)                                                    FET05450
      IF(LTEMP.EQ.0)GO TO 70                                            FET05460
      DO 72  K=1,7,1                                                    FET05470
      KTEMP=LIDS(I,K)                                                   FET05480
      IF(KTEMP.EQ.0) GO TO 71                                           FET05490
      ON=O(KTEMP)                                                       FET05500
      PN=P(KTEMP)                                                       FET05510
      QN=Q(KTEMP)                                                       FET05520
      M=1                                                               FET05530
      IF(LID(I,J,M).NE.0) M=2                                           FET05540
      IF(ON.EQ.LTEMP.OR.PN.EQ.LTEMP.OR.QN.EQ.LTEMP)LID(I,J,M)=KTEMP     FET05550
   72 CONTINUE                                                          FET05560
   71 CONTINUE                                                          FET05570
   70 CONTINUE                                                          FET05580
C     LET A(I,J)=0,MAP(I,J)=0,B(I)=0                                    FET05590
      DO 77 I = 1, IT, 1                                                FET05600
      DO 78 J = 1, 16, 1                                                FET05610
      A(I,J)=0.                                                         FET05620
      MAP(I,J)=0                                                        FET05630
   78 CONTINUE                                                          FET05640
      B(I)=0.                                                           FET05650
   77 CONTINUE                                                          FET05660
C      ITERATION ON I=NCELL IS TO FORM YC(..),XC(..)                    FET05670
      DO 85 I = 1, NCELL, 1                                             FET05680
      ON=O(I)                                                           FET05690
      PN=P(I)                                                           FET05700
      QN=Q(I)                                                           FET05710
      YC(I,1)=Y(PN)-Y(QN)                                               FET05720
      YC(I,2)=Y(QN)-Y(ON)                                               FET05730
      YC(I,3)=Y(ON)-Y(PN)                                               FET05740
      XC(I,1)=X(QN)-X(PN)                                               FET05750
      XC(I,2)=X(ON)-X(QN)                                               FET05760
      XC(I,3)=X(PN)-X(ON)                                               FET05770
      AREN(I)=.5*(XC(I,1)*YC(I,3)-XC(I,3)*YC(I,1))                      FET05780
   85 CONTINUE                                                          FET05790
C                                                                       FET05800
C                         *************************                     FET05810
C                         *  FORM STIFFNESS MATRIX *                    FET05820
C                         *************************                     FET05830
C                                                                       FET05840
      DO 90 I=1,NODE,1                                                  FET05850
      IV=NODE+I                                                         FET05860
      DO 91  JA=1,8,1                                                   FET05870
      J=MID(I,JA)                                                       FET05880
      JB=JA+8                                                           FET05890
      LD=0                                                              FET05900
      DO 92 KA=1,7,1                                                    FET05910
      LD=LD+1                                                           FET05920
      IF(I.EQ.J) GO TO 94                                               FET05930
```

```
      IF(LD.GT.2) GO TO 92                                              FET05940
      K=LID(I,JA,KA)                                                    FET05950
      GO TO 95                                                          FET05960
   94 K=LIDS(I,KA)                                                      FET05970
   95 IF(K.EQ.0) GO TO 92                                               FET05980
      AREA=ABS(AREN(K))                                                 FET05990
      IH=I                                                              FET06000
      DO 97 IL=1,2,1                                                    FET06010
      IF(O(K).EQ.IH) KJ=1                                               FET06020
      IF(P(K).EQ.IH) KJ=2                                               FET06030
      IF(Q(K).EQ.IH) KJ=3                                               FET06040
      YJ=YC(K,KJ)                                                       FET06050
      XJ=XC(K,KJ)                                                       FET06060
      IF(IL.EQ.2) GO TO 97                                              FET06070
      IH=J                                                              FET06080
      XI=XJ                                                             FET06090
      YI=YJ                                                             FET06100
   97 CONTINUE                                                          FET06110
      AT1=(2.*YI*YJ*(.5*  ELAM(K)+  EMU(K))+XI*XJ*  EMU(K))/(4.*AREA)  FET06120
      AT2=(YI*XJ*  ELAM(K)+XI*YJ*  EMU(K))/(4.*AREA)                    FET06130
      AT3=(XI*YJ*  ELAM(K)+YI*XJ*  EMU(K))/(4.*AREA)                    FET06140
      AT4=(2.*XI*XJ*(.5*  ELAM(K)+  EMU(K))+YI*YJ*  EMU(K))/(4.*AREA)  FET06150
      IF(IGEOM.EQ.1) GO TO 98                                           FET06160
C                                                                       FET06170
C     ADDITIONS TO MATRIX FOR AXI-SYMMETRIC                             FET06180
C                                                                       FET06190
      ON=O(K)                                                           FET06200
      PN=P(K)                                                           FET06210
      QN=Q(K)                                                           FET06220
      XAV=(X(ON)+X(PN)+X(QN))/3.                                        FET06230
       AT1=(AT1*3.1416*2.*XAV)+ (4.*3.1416*AREA/(9.*XAV))*(EMU(K)+      FET06240
     1 .5*ELAM(K))+(((AREA*3.1416*ELAM(K))*(YI+YJ))/(3.*AREN(K)))       FET06250
       AT2=(AT2*3.1416*2.*XAV)+(3.1416*AREA*ELAM(K)*XJ)/(3.*AREN(K))    FET06260
       AT3=(AT3*3.1416*2.*XAV)+(3.1416*AREA*ELAM(K)*XI)/(3.*AREN(K))    FET06270
       AT4=AT4*3.1416*2.*XAV                                            FET06280
   98 A(I,JA)=A(I,JA)+AT1                                               FET06290
      A(I,JB)=A(I,JB)+AT2                                               FET06300
      A(IV,JA)=A(IV,JA)+AT3                                             FET06310
      A(IV,JB)=A(IV,JB)+AT4                                             FET06320
   92 CONTINUE                                                          FET06330
      MAP(I,JA)=J                                                       FET06340
      MAP(I,JB)=J+NODE                                                  FET06350
      MAP(IV,JA)=J                                                      FET06360
      MAP(IV,JB)=J+NODE                                                 FET06370
   91 CONTINUE                                                          FET06380
   90 CONTINUE                                                          FET06390
C                                                                       FET06400
C                **************************                             FET06410
C                * ADD SPECIFIED LOADS TO B(I) *                        FET06420
C                **************************                             FET06430
```

```
C
      DO 100 I=1,NODE,1
      IV=NODE+I
      B(I)=FX(I)
      B(IV)=FY(I)
  100 CONTINUE
C     ARRANGE MATRIX FOR SOLUTION
      DO 101 I=1,IT,1
      DO 102 J = 1, 16, 1
      IF (I.EQ.MAP(I,J)) GO TO 103
  102 CONTINUE
  103 AR(I)=A(I,J)
  101 CONTINUE
C
C     SOLVE SIMULTANEOUS EQUATIONS
C
      M=0
      DO 104 L=1,ITERAT,1
      DO 105 I=1,IT,1
      IF(NDISP(I).EQ.1) GO TO 105
      TEMP=0.
      DO 106 J=1,16,1
      IF(MAP(I,J).EQ.0) GO TO 106
      IF (I.EQ.MAP(I,J)) GO TO 106
      JROW=MAP(I,J)
      TEMP=TEMP+A(I,J)*DISP(JROW)
  106 CONTINUE
      DISP(I)=((B(I)-TEMP)/AR(I))*W+((1.-W)*DISP(I))*10.**NC
      IF(DISP(I).GT.1.0E07) GO TO 825
  105 CONTINUE
      IF(L.NE.25.AND.L.LE.49)GO TO 104
      ITL=L/25
      ITLL=25*ITL
      IF(ITLL.NE.L)GO TO 901
906   M=M+1
      GO TO (15,16,2001),M
15    DIST(M)=SQRT((DISP(MIS))**2+(DISP(MIDD))**2)
      GO TO 104
16    DIST(M)=SQRT((DISP(MIS))**2+(DISP(MIDD))**2)
      PERC=SQRT((DIST(M-1)-DIST(M))**2)/DIST(M-1)
      DIST(M-1)=DIST(M)
      M=M-1
901   IF(PERC.LE.CUT)GO TO 902
  104 CONTINUE
902   M=2
904   WRITE(6,163) L
      IF(IPO.EQ.8)GO TO 2
      WRITE(6,110)
      WRITE(6,111)
2     NOD=NODE
```

```
      IF(IPO.EQ.5.OR.IPO.EQ.7)NOD=NF
      DO 112 I=1,NOD,1
       IV=NODE+I
      DISP(I)=DISP(I)-1.0
      DISP(IV)=DISP(IV)-1.0
      IF(IPO.EQ.8)GO TO 112
       WRITE(6,113) I,DISP(I),DISP(IV),X(I),Y(I)
  112  CONTINUE
      DO 1111 I=1,NF,1
      IV=NODE+I
 1111 XPLOT(I)=DISP(IV)
      CALL QPLOT(LINES,LPLOT,XPLOT,FILL,IIHR,NPT,RNG,DMIN,DAY1)
      IF(IGEOM.EQ.1.AND.M.EQ.2)GO TO 2001
      GO TO 120
  825 CONTINUE
      WRITE(6,830)
      WRITE(6,831)
      DO 826 I=1,NODE,1
      WRITE(6,13) I,X(I),Y(I)
  826 CONTINUE
      STOP
  120 CONTINUE
C
C        BYPASS LOOP FOR STRESS STRAIN CALCULATIONS
C     * CALCULATE STRESSES AND STRAINS*
C
      IF (IGEOM.EQ.1) GO TO 2001
      WRITE(6,210)
  210 FORMAT(6X,71H************* STRESS AND STRAIN IN PLANE OF CROSS-SEC
     1TION *************)
      WRITE(6,211)
  211 FORMAT(6X,1H*,69X,1H*)
      WRITE(6,212)
  212 FORMAT(1X,127HCELL * STRESS MAX.  STRESS MIN.     STRAIN      STRAI
     1N      ANGLE     ANGLE    *    HOOP          HOOP      SHEAR STRESS
     2 PRESSURE)
      WRITE(6,213)
  213 FORMAT(6X,107H*                                    MAX.        MIN.
     1  MAX        MIN      *   STRESS       STRAIN      (MAXIMUM))
      DO 200 I=1,NCELL,1
      OE=O(I)
      PE=P(I)
      QE=Q(I)
      UO=DISP(OE)
      UP=DISP(PE)
      UQ=DISP(QE)
      OE=OE+NODE
      PE=PE+NODE
      QE=QE+NODE
      VO=DISP(OE)
```

```
      VP=DISP(PE)                                                       FET07440
      VQ=DISP(QE)                                                       FET07450
      EX=(UO*YC(I,1)+UP*YC(I,2)+UQ*YC(I,3))/(2.*AREN(I))                FET07460
      EY=(VO*XC(I,1)+VP*XC(I,2)+VQ*XC(I,3))/(2.*AREN(I))                FET07470
      EXY=(UO*XC(I,1)+UP*XC(I,2)+UQ*XC(I,3)+VO*YC(I,1)+VP*YC(I,2)+VQ*YC(FET07480
     1I,3))/(4.*AREN(I))                                                FET07490
      AE=EX-EY                                                          FET07500
      BE=EX+EY                                                          FET07510
      AEP=ABS(AE)                                                       FET07520
      IF(AEP.LT..00001) GO TO 201                                       FET07530
      EBETA=ATAN((2.*EXY)/AE)/2.                                        FET07540
      GO TO 202                                                         FET07550
  201 EBETA=3.14159/4.                                                  FET07560
  202 EGAM=EBETA+3.14159/2.                                             FET07570
      EP1=(BE/2.)+(AE/2.)*COS(2.*EBETA)+EXY*SIN(2.*EBETA)               FET07580
      EP2=(BE/2.)+(AE/2.)*COS(2.*EGAM)+EXY*SIN(2.*EGAM)                 FET07590
      IF(EP1.LT.EP2) GO TO 203                                          FET07600
      EMAX=EP1                                                          FET07610
      EMIN=EP2                                                          FET07620
      THETA1=EBETA                                                      FET07630
      THETA2=EGAM                                                       FET07640
      GO TO 204                                                         FET07650
  203 EMAX=EP2                                                          FET07660
      EMIN=EP1                                                          FET07670
      THETA1=EGAM                                                       FET07680
      THETA2=EBETA                                                      FET07690
  204 THETA1=THETA1*57.2958                                             FET07700
      THETA2=THETA2*57.2958                                             FET07710
      DELV=(EMAX+EMIN)/2.                                               FET07720
      SIGMAX=2.*EMU(I)*EMAX+ELAM(I)*2.*DELV                             FET07730
      SIGMIN=2.*EMU(I)*EMIN+ELAM(I)*2.*DELV                             FET07740
      PRESS=(SIGMAX+SIGMIN)/2.                                          FET07750
      SHEAR=(SIGMAX-SIGMIN)/2.                                          FET07760
      HOOPE=0.                                                          FET07770
      HOOPS=0.                                                          FET07780
      IF(IGEOM.EQ.1) GO TO 205                                          FET07790
      OE=OE-NODE                                                        FET07800
      PE=PE-NODE                                                        FET07810
      QE=QE-NODE                                                        FET07820
      HOOPE=(UO+UP+UQ)/(X(OE)+X(PE)+X(QE))                              FET07830
      DELV=(HOOPE+EMAX+EMIN)/3.                                         FET07840
      SIGMAX=2.*EMU(I)*EMAX+ELAM(I)*3.*DELV                             FET07850
      SIGMIN=2.*EMU(I)*EMIN+ELAM(I)*3.*DELV                             FET07860
      PRESS=(SIGMAX+SIGMIN+HOOPS)/3.                                    FET07880
      HOOPS=2.*EMU(I)*HOOPE+ELAM(I)*3.*DELV                             FET07870
      IF(HOOPS.LT.SIGMAX.AND.HOOPS.GT.SIGMIN) SHEAR=(SIGMAX-SIGMIN)/2.  FET07890
      IF(HOOPS.GT.SIGMAX) SHEAR=(HOOPS-SIGMIN)/2.                       FET07900
      IF(HOOPS.LT.SIGMIN) SHEAR=(SIGMAX-HOOPS)/2.                       FET07910
  205 WRITE(6,214)I,SIGMAX,SIGMIN,EMAX,EMIN,THETA1,THETA2,HOOPS,HOOPE,  FET07920
     1SHEAR,PRESS                                                       FET07930
```

```
 214 FORMAT(2X,I3,3X,E11.4,2X,E11.4,2X,E11.4,1X,E11.4,1X,F7.2,2X,F7.2,  FET07940
    13X,E11.4,2X,E11.4,3X,E11.4,4X,E11.4)                              FET07950
     SIGX=2.*EMU(I)*EX+ELAM(I)*(EX+EY)                                 FET07960
     SIGY=2.*EMU(I)*EY+ELAM(I)*(EX+EY)                                 FET07970
     SIGXY=2.*EMU(I)*EXY                                               FET07980
     WRITE(6,950) SIGX,SIGY,SIGXY                                      FET07990
 950 FORMAT(12X,E11.4,2X,E11.4,2X,E11.4)                               FET08000
 200 CONTINUE                                                          FET08010
2001 CONTINUE                                                          FET08020
     RETURN                                                            FET08030
     END                                                               FET08040
```

```
      SUBROUTINE QPLOT (LINES,L,X,FILL,IIHR,NPT,RNG,DMIN,DAY1)          FET08060
C                                                                       FET08070
C     PRINTER PLOT OF DISPLACEMENTS OF SURFACE NODES                    FET08080
C                                                                       FET08090
      DIMENSION X(L),LINE(127),ICHAN(10),CMNT(20)                       FET08100
      DIMENSION IAXIS(100)                                              FET08110
      DATA IBLANK,IIII/'    ','IIII'/                                   FET08120
      DATA ICHAN/'....','NNNN','0000','4444','5555','6666','7777',      FET08130
     1'7777','8888','9999'/                                             FET08140
      DATA IDOT,IDASH/'.','|'/                                          FET08150
      DO 32 I=1,10                                                      FET08160
      DO 34 J=1,9                                                       FET08170
   34 IAXIS(10*(I-1)+J)=IDOT                                            FET08180
   32 IAXIS(10*I)=IDASH                                                 FET08190
      IF(NPT.EQ.8)GO TO 22                                              FET08200
      WRITE(6,100)                                                      FET08210
  100 FORMAT(1H1)                                                       FET08220
22    WRITE(6,12)                                                       FET08230
   12 FORMAT(1H0)                                                       FET08240
      WRITE(6,11) (IAXIS(JJ),JJ=1,100)                                  FET08250
   11 FORMAT(28X,100A1)                                                 FET08260
      DO 6 I=1,L,1                                                      FET08270
      IF(X(I).GE.0)GO TO 6                                              FET08280
      IIHR=IIHR+1                                                       FET08290
      IF(IIHR.EQ.1)GO TO 7                                              FET08300
      DMIN=X(I)                                                         FET08310
      IF(DMIN.GE.RNG)GO TO 6                                            FET08320
      RNG=DMIN                                                          FET08330
      GO TO 6                                                           FET08340
7     RNG=X(I)                                                          FET08350
6     CONTINUE                                                          FET08360
      DO 13 I=1,L,1                                                     FET08370
      X(I)=X(I)-RNG                                                     FET08380
13    CONTINUE                                                          FET08390
      B=0.0                                                             FET08400
      DO 5 I=1,L,1                                                      FET08410
5     IF(X(I).GT.B) B=X(I)                                              FET08420
      DO 30 K=1,L,1                                                     FET08430
      DO 10 I=1,100,1                                                   FET08440
10    LINE(I)=IBLANK                                                    FET08450
      INTEG=1.0+X(K)*FILL/B                                             FET08460
      IF(INTEG.GT.100) INTEG=100                                        FET08470
      LINE(DAY1)=IIII                                                   FET08480
      LINE(INTEG)=ICHAN(1)                                              FET08490
      X(K)=X(K)+RNG                                                     FET08500
      WRITE(6,43) K,X(K),(LINE(I),I=1,100)                              FET08510
   43 FORMAT(1X,I3,2X,E11.4,11X,100A1)                                  FET08520
30    CONTINUE                                                          FET08530
      WRITE(6,12)                                                       FET08540
      WRITE(6,11) (IAXIS(JJ),JJ=1,100)                                  FET08550
```

```
      WRITE(6,205)                                                      FET08560
  205 FORMAT(1H1)                                                       FET08570
      RETURN                                                            FET08580
      END                                                               FET08590
```

```
C
      FOLLOWING IS BLOCK DATA ARRAY.  IT IS ALWAYS THE FIRST FIFTEEN
      CARDS IN DATA DECK. BLOCK ARRAY ALWAYS REMAINS THE SAME REGARDLESS
      OF THE NUMBER OR TYPE OF MODELS TO BE PROCESSED.
      THE FORMAT OF THE BLOCK ARRAY IS 10F5.1

  2.0  2.0  2.0  2.0  2.0  2.0  0.0  0.0  0.0  4.0                       1
  0.2  1.0  2.0  2.0  3.0  4.0  0.0  0.0  0.0  0.0                       2
  1.0  1.0  1.0  1.0  1.0  2.0  0.0  0.0  0.0  1.0                       3
  1.0  1.0  1.0  1.0  1.0  1.0  0.0  0.0  0.0  1.0                       4
  0.2  1.0  2.0  2.0  3.0  4.0  0.0  0.0  0.0  0.0                       5
  1.0  1.0  3.0  3.0  1.0  1.0  0.0  0.0  0.0  2.0                       6
  1.0  1.0  1.0  1.0  1.0  1.0  1.0  2.0  0.0  4.0                       7
  0.2  1.0  1.0  1.0  1.0  1.0  3.0  4.0  0.0  0.0                       8
  1.0  1.0  1.0  1.0  1.0  1.0  2.0  2.0  0.0  3.0                       9
  0.5  0.5  0.5  0.5  0.5  0.5  1.0  2.0  0.0  4.0                      10
  0.2  1.0  1.0  1.0  1.0  1.0  3.0  4.0  0.0  0.0                      11
  1.0  1.0  1.0  1.0  1.0  2.0  2.0  2.0  0.0  4.0                      12
  1.0  1.0  1.0  1.0  1.0  1.0  0.0  0.0  0.0  1.0                      13
  0.2  1.0  2.0  2.0  3.0  4.0  0.0  0.0  0.0  0.0                      14
  1.0  1.0  4.0  4.0  1.0  1.0  0.0  0.0  0.0  5.0                      15
```

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Thesis
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c.1
Modeling an input-output geokinetic system utilizing a finite element approach.